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Price Level Targeting

Optimal anchoring of expectations in a New Keynesian model

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Executive summary

The recent experience of the *Financial Crisis* has highlighted the potential drawbacks of a policy targeting the *changes* and not the *level* of the prices. When the zero lower bound on the nominal interest rate binds, the inflation target presents a lower constraint on the real interest rate because inflation expectations are anchored at the target. Following the crisis, a number of major central banks have been forced to keep the policy rate close to zero and in the mean time use unconventional tools to keep monetary policy effective. Price level targeting, however, presents the optimal way of anchoring expectations by increasing inflation expectations in a deflationary environment and *vice versa*. This improves monetary policy in general and in a zero interest rate environment, which keeps the conventional interest rate operating procedure effective. This thesis attempts to study, how the central bank can optimally utilise the expectational channel, when setting monetary policy, by announcing a price level target. The investigation will take on both a theoretical and an empirical stand point. The theoretical part of the thesis revisits the arguments for and against adopting price level targeting. The empirical part of the thesis attempts to evaluate the optimality of monetary policy by inspecting the statistical properties of the price level.

The first part of the thesis introduces a New Keynesian model with forward-looking rational economic agents. Because it is assumed to be costly for firms to change prices, and because households are assumed to smooth consumption, the current state of the economy in the model depends on expectations about the future. Due to the existence of this property in the model, the central bank can affect the current state of the economy by influencing the private sector's expectations about the future. Because society is concerned with deviations in inflation and the output gap, and because this thesis looks at monetary policy, when the central bank is concerned about deviations in the price level, the assumptions, necessary for a central bank to have different preferences for monetary policy than society, are also discussed.

Second, the thesis shows that the optimal commitment solution to monetary policy, when expectations are forward-looking, is history dependent and involves a stationary price level. The optimal commitment solution improves monetary policy compared to the solution under discretion by optimally utilising the forward-looking expectations. The impulse response to a temporary cost-push shock is used to illustrate the difference. A central bank acting under discretion makes the entire adjustment immediately, while a central bank making the optimal commitment promises to deflate the economy. Hence, the optimal commitment policy does not allow shocks to the price level to persist and bygones are therefore not bygones. Consequently, the improvement is a result of the private sector taking this reaction into account when forming expectations which in turn reduces the effect of the shock on the current variables. Thus, a central bank making the optimal commitment is letting the market do some of the stabilisation. However, because commitment remains an obstacle for the central bank, a number of alternative policies, which implement a history dependent policy without the central bank having to commit, are briefly reviewed. These include a policy which targets nominal income growth and a policy that targets the change in the output gap.

The third part focuses on a third alternative policy, which targets the price level. This policy is particularly interesting because it replicates both the salient feature of history dependence and includes a stationary price level, which are the main characteristics of the optimal commitment solution. In fact, it is shown that when there is no persistence in cost-push shocks, then it is possible to perfectly replicate the optimal commitment solution by assigning a price level target to the central bank. It is then further shown, how price level targeting presents a way of keeping the conventional interest rate operating procedure effective, when the zero lower bound binds. When the price level undershoots the target, the private sector expects inflation in the future which lowers the real interest rate and adds stimulus to the economy. However, if an escape clause, which allows the central bank to ignore certain shocks to the price level and adopt the response under inflation targeting, is added to the policy, then the stabilising effect through expectations is limited. Finally, the implications, when the price level targeting policy lacks credibility, are investigated. It turns out, that even though it may involve some transitional costs adopting a price level target because the private sector first has to learn about the policy, it is still optimal in the long-run to implement the policy.

The fourth and final part of the thesis conducts an empirical investigation of the optimality of price level targeting based on the theoretical results. The conventional way of evaluating monetary policy is by inspecting the objective for monetary policy announced by the central bank. However, this thesis uses a different approach. Because optimal monetary policy involves a stationary price level, the optimality of monetary policy is evaluated by inspecting the statistical properties of the price level in ten major countries. It turns out that the central bank in seven out of the ten countries has set a stationary price level. These include the central banks in Australia, Canada, Japan, New Zealand, Norway, Switzerland and US. In contrast, a review of the ten central bank objectives reveals that none of them have announced an objective which, formally, is equivalent with such policy. The central banks in Euro Area, Sweden and UK have not set a stationary price level. The latter three central banks may therefore improve on monetary policy by adopting price level targeting, while the recommendation to the former seven depends on how expectations are formed.

In summary, the overall conclusion of this thesis is that price level targeting presents a way to implement the optimal commitment solution to monetary policy, when the central bank is forced to act under discretion and thus a way to improve monetary policy compared to discretionary inflation targeting. Furthermore, price level targeting has additional leverage over inflation targeting when the zero lower bound binds. However, if the central bank is allowed to adopt the response under inflation targeting and ignore certain shocks to the price level, then the benefits of price level targeting is limited. It is therefore important that the central bank only "targets what it can hit". Additionally, price level targeting remains beneficial in the long-run even though it may involve short-run costs as the private sector learns about the policy. Empirically, the central banks in Australia, Canada, Japan, New Zealand, Norway, Switzerland and US are found to have set a stationary price level and thus a policy which resembles the optimal commitment. The central banks in Euro Area, Sweden and UK are, however, not found to have set a stationary price level. The latter three central banks may therefore improve on monetary policy by adopting price level targeting.

Preface

During my studies at Department of Economics at University of Copenhagen I have gained an interest in the field of macroeconomics. After following the course Monetary Economics: Macro Aspects I further gained a particular interest in monetary policy. Through my position as an analyst in Danske Research in Danske Bank, I have learned about practical monetary policy. I have found an interest in studying ways for the central bank to improve monetary policy by better utilising expectations and this interest has grown stronger following the Financial Crisis, which has challenged the conventional knowledge about monetary policy. When I attended the seminar Advanced Monetary Macro, I wrote a paper within this topic. The paper and the seminar discussions further helped inspire me to write this thesis.

First of all, I am grateful for the support offered, through valuable discussions and comments, by my supervisor, Henrik Jensen, professor at Department of Economics, University of Copenhagen. Beside from this, I would like to thank Niels Blomquist, economist at the Danish National Bank, Louise Aggerstrøm Hansen, graduate student at Department of Economics, University of Copenhagen and Rune Juhl, graduate student at Technical University of Denmark for insightful and useful comments and their great effort in proofreading. Any remaining errors are entirely my own responsibility.

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"Instead, in a decade or so, when central banks (hopefully) master maintaining low an stable inflation, the time may be ripe for seriously considering price level stability as a goal for monetary policy.", Svensson (1999a) p. 1

1 Introduction

The emergence of inflation targeting has helped central banks master maintaining a low and stable level of inflation. There is a general consensus in the literature that current economic activity depends on expectations about the future. In this context inflation targeting has become an effective way of anchoring inflation expectations which in turn has enabled the central bank to better control the real interest rate. However, when the zero lower bound on the nominal interest rate binds, the inflation target presents a lower constraint on the real interest rate. The recent experience of the *Financial Crisis* highlights the potential drawbacks of this policy. A number of major central banks have been forced to keep the policy rate close to zero since the crisis broke out and in the mean time turned to the use of unconventional tools in order to keep monetary policy effective. Price level targeting, however, presents the optimal way of anchoring inflation expectations. This can improve monetary policy compared to inflation targeting and furthermore, keep the conventional interest rate operating procedure effective when the zero lower bound binds.

Conventionally, price level targeting has been viewed as advantageous over inflation targeting in the long-run as it induces greater price level certainty, but only at the cost of higher volatility in short-run. However, when economic agents have forward-looking expectations about the future, a target for the price level presents the optimal way to utilise these expectations when setting monetary policy. Svensson (1999b) labelled this result a "free lunch" for monetary policy and Vestin (2006) has proved that price level targeting constitutes optimal monetary policy.

So far, the "conventional wisdom" seems to have prevailed though. No central bank is currently operating a price level targeting regime. However, Bank of Canada has since 2006 actively been considering the possibility of adopting a price level target. Hence, this may be an early indication that central banks are gaining courage to implement the policy. Should a central decide to adopt a target for the price level, it would only be the second central bank to ever do so. In Sweden in the 1930s the Riksbank became the first central bank to adopt price level targeting. However, with the effectiveness of current monetary policy challenged by the Financial Crisis, the time may be ripe for central banks to consider price *level* stability as a goal for monetary policy.

This thesis will investigate the implications of adopting a policy focused on a price level target. This will be done from a theoretical and an empirical point of view. The main focus of the theoretical investigation will be on optimal monetary policy in the well-known New Keynesian model with rational forward-looking private agents. The thesis attempts to study how the central bank can optimally utilise the expectational channel when setting monetary policy. A large strain of literature has found policies which set a stationary price level to optimally utilise the private sector's expectations and furthermore price level targeting to be one policy which achieves this. This thesis contributes to the existing literature by revisiting the different arguments for and against adopting price level targeting. The main focus of the empirical investigation is on the application of the theoretical results when evaluating monetary policy. The conventional way of evaluating the optimality of monetary policy is to analyse the objective for monetary policy announced by the central bank. This thesis contributes to the existing knowledge on empirical monetary policy by applying a different method of evaluating monetary policy which attempts to make conclusions about the optimality of monetary policy by inspecting the statistical properties of the price level.

The remaining thesis is organised as follows: in section 2, a New Keynesian model is introduced. Because of the great importance of the private sector's expectations on the conduct of monetary policy, a particular emphasis is put on discussing the assumptions implying a forward-looking private sector. Section 3 then shows that the optimal commitment solution to monetary policy is history dependent and includes a stationary price level. To clarify the characteristics of a history dependent policy, the solution is compared to a purely forward-looking policy. The central bank is generally not assumed to be able to commit. The problems with commitment and potential ways of easing commitment are therefore analysed along with alternative ways of implementing the optimal commitment solution, when central bank is forced to act under discretion. In section 4, the specific case of price level targeting is analysed. The section first shows how the optimal commitment solution can be replicated using price level targeting. The section then looks at some important issues regarding price level targeting, which include the additional advantages of price level targeting, when the zero lower bound on the nominal interest rate binds, the implications of using an escape clause to ignore past shocks and the favourability of the policy if it lacks perfect credibility. Section 5 applies the theoretical results about optimal monetary policy in an empirical investigation. Using a broad sample of major central banks, the section first reviews what objectives for monetary policy the central banks announce. Then, using the result that optimal monetary policy involves a stationary price level, the section investigate whether past monetary policy has resembled the optimal commitment and finally, the implications of the empirical findings for future monetary policy are discussed. Section 6 concludes.

2 New Keynesian model

The model used throughout the paper to analyse different aspects of monetary policy belongs to the so-called *New Keynesian framework*. The variant used here is a small-scale closed-economy model which builds on the principles of chapter 3 and 6 in Woodford (2003), chapter 3 and 4 in Galí (2008) and chapter 8 in Walsh (2010). One important aspect of the model is that it is based on a micro foundation which makes it robust to the *Lucas critique* in Lucas (1976). Unless otherwise noted, all variables are expressed in logarithmic terms. The model describes an economy of optimising agents. The economy consists of a private sector and a central bank. The following section will qualitatively motivate the key aggregate relationships of the model. Technical details of the underlying micro foundation are available in appendix 8.1.

2.1 The private sector

The households in the economy identically supply labor, hold money and consume a basket of goods based on utility maximisation, while identical firms hire labor and produce differentiated goods based on monopolistic competitive profit maximisation. For simplicity, capital and investments are ignored in the model.

The aggregate supply side of the economy is modelled as an expectations augmented *Phillips curve*. The Phillips curve arrives from the assumption of monopolistic competition in the goods market and an assumption of staggered price setting. This means that a firm sets an individual price of a differentiated produced good which maximises the firm's profit, while realising that the firm is not able to adjust prices freely in the future.

Current inflation is affected through three different channels. First, current inflation depends on expected future inflation. This part can be contributed to the assumption of staggered prices. The staggered price setting used to derive the Phillips curve is of the type introduced in Calvo (1983). In a *Calvo model* of staggered price setting it is assumed that in every period a fraction, $1-\omega$, of firms can adjust their prices, while the remaining fraction, $0 \leq \omega \leq 1$, are forced to remain with their current prices. This form of price setting can be justified by costly price changes due to, for example, menu costs. The special case of $\omega = 0$ corresponds to a situation of flexible prices. The opportunity to change price occurs randomly. A firm will, when it is given the opportunity to change its individual price, set the price according to

$$min\ \frac{1}{2}E_t \sum_{j=0}^{\infty} \beta^j \left(p_{i,t+j} - p^{\star}_{t+j}\right)^2$$

conditional on when the firm expects to change its price again. Hence, the firm will seek to minimise any expected future deviations between the actual price, $p_{i,t+j}$, it charges and the optimal profit maximising price, $p^{\star}_{t+j}$, it would charge in the absence of any restrictions or adjustment costs.[1] This establishes a connection between current prices and expected future prices.

Moving on from the Calvo model, the nature of monopolistic competition on the goods market imply that firms will set $p^{\star}$ as a mark-up over real marginal costs. Chapter 8 in Walsh (2010) and appendix 8.1 show that real marginal costs are proportional to the output gap, x, which then establishes the second channel where current inflation depends on the output gap. The third channel adds exogenous disturbance to current inflation. This reflects movements in real marginal costs that are independent of the movements in real marginal costs captured by the output gap term, while further enabling the possibility of exogenous persistence in the inflation process.

Staggered price setting, monopolistic competition and exogenous disturbance lead to the following expression of the *New Keynesian Phillips curve* used in this model

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + u_t \tag{2.1}$$

Equation (2.1) shows how current inflation, defined as $\pi_t \equiv p_t - p_{t-1}$, depends on expected future inflation, $E_t \pi_{t+1}$, discounted by the factor, $0 < \beta < 1$, the current output gap, x_t, and the exogenous disturbance, u_t. The output gap is defined as $x_t \equiv y_t - y_t^f$, where y_t^f is potential output, which is obtained in an economy with flexible wages and prices. The parameter, κ, in front of the output gap depends positively on the fraction of firms able to adjust prices every period. Hence, when a large fraction of firms is able to adjust prices every period κ is large, which means current prices, to a larger extent, will reflect current real marginal costs. Consequently, current inflation will be more dependent on the current output gap. Furthermore, κ depends negatively on the discount factor. This is because a high discount factor means that firms place a higher weight on future profits which then implies that current real marginal costs have less impact on current price setting. Finally, the disturbance term follows $u_t = \rho u_{t-1} + e_t$, where e_t is assumed to be a random i.i.d. variable with mean zero and constant variance, σ_e^2 and $\rho \in [0; 1]$. u_t can be interpreted as a cost-push shock.[2] A positive cost-push shock is thus a shock to real marginal costs which pushes up inflation. $\rho > 0$ adds exogenous persistence to the inflationary process. The nominal rigidities enable an active role for monetary policy in the short run. The New Keynesian Phillips curve has furthermore found empirical support.[3]

[1] As further explained in appendix 8.1 all firms are essentially identical and the subscript, i, has thus been left out of the notation.

[2] Galí (2008), for example, formally shows how the exogenous disturbance term may be interpreted as the deviation in the efficient level of output from potential output.

[3] Galí and Gertler (1999) and Galí et al. (2005) show that a hybrid version of equation (2.1), which allows for a fraction of the firms to exhibit a rule-of-thumb price setting behavior, does well in explaining

The aggregate demand side of the economy is modelled as an *IS* type relation. It results from utility maximising households. In this model household utility is defined over a composite consumption good, real money balances and leisure. Imposing the closed economy resource constraint that consumption equals output, the solution to the household optimisation problem is the consumption Euler equation expressed by the well-known *Keynes-Ramsey rule* - see appendix 8.1.

By log-linearising the Euler equation the following aggregate demand relation arrives

$$x_t = E_t x_{t+1} - \frac{1}{\sigma}\left(i_t - E_t \pi_{t+1}\right) + g_t \tag{2.2}$$

which links the current output gap to the expected future output gap, $E_t x_{t+1}$, the current real interest rate, $i_t - E_t \pi_{t+1}$, and the exogenous disturbance, g_t. The positive relationship between the current and future output gap can be contributed the preference for consumption smoothing among households. If the households expect a rise in future demand, hence, a future rise in consumption, they will raise current consumption and demand because they prefer to smooth consumption. The negative relationship between the current real interest rate and current output gap is contributed intertemporal substitution of consumption. The parameter, $\frac{1}{\sigma} > 0$, is the intertemporal elasticity of substitution. The exogenous disturbance term evolves according to $g_t = \mu g_{t-1} + a_t$, where a_t is assumed to be an i.i.d. random variable with mean zero and constant variance σ_a^2 and $\mu \in [0; 1]$.[4]

2.1.1 Monetary transmission and forward-looking expectations

The central bank is assumed to use the short term nominal interest rate to set monetary policy. This is the most common operating procedure in modern central banking. The quantity of money has no explicit role in this model. The central bank will set the quantity of money endogenously to achieve equilibrium in the money market corresponding to the desired nominal interest rate. Sticky prices imply that monetary policy has leverage over the real interest rate in the short run. Furthermore, monopolistic competition implies that current inflation depends on the current output gap. Hence, monetary policy is not neutral in the short run in this model.

From equations (2.1) and (2.2) monetary policy therefore works through the following transmission mechanism: following a shock to the economy, the central bank adjusts the short term nominal interest rate to offset the shock. Due to sticky prices, this changes

the inflation dynamics in the US. Smets and Wouters (2003) find that the New Keynesian Phillips curve does well in describing the inflation dynamics in the Euro Area.

[4]From appendix 8.1, $g_t \equiv \left(E_t y_{t+1}^f - y_t^f\right) + \frac{\sigma-1}{\sigma}\left(E_t \varpi_{t+i}^C - \varpi_t^C\right)$, where $E_t \varpi_{t+i}^C - \varpi_t^C$ is expected changes to the household preferences for consumption. Even though the disturbance term g_t is augmented on the demand side of the economy it adds a supply side dimension through the inclusion of the expected change in potential output, $E_t y_{t+1}^f - y_t^f$.

the real interest rate, which in turn affects aggregate demand and, thus, the output gap. Finally, the change in the output gap affects inflation.

However, monetary policy does not only work through changing the nominal interest rate. Monetary policy can also affect the economy by affecting the private sector's expectation about the future. In this model firms set prices and households make decisions on consumption based on expectations about the future economic development. To fully illustrate this property of the model the Phillips curve and the aggregate demand relation are iterated forward.

Iterating equation (2.1) forward leads to the following relation

$$\pi_t = E_t \sum_{i=0}^{\infty} \beta^i \left(\kappa x_{t+i} + u_{t+i}\right) \tag{2.3}$$

This highlights how the firms set prices to meet current real marginal costs and discounted future real marginal costs. And as mentioned before this is proportional to expected future economic conditions.

Iterating equation (2.4) forward leads to this relation

$$x_t = E_t \sum_{i=0}^{\infty} \left\{ -\frac{1}{\sigma} \left(i_{t+i} - \pi_{t+1+i}\right) + g_{t+i} \right\} \tag{2.4}$$

which shows how current demand from households depends on the current real interest rate and the expected future path of the real interest rate. Hence, monetary policy does not only work through the current real interest rate, but also through the expected future real interest rate and, thus, the expected path of both the nominal interest rate and inflation.

It is this point which is of interest for the analysis in this thesis. The central bank should not only pay attention to the effects of its current actions, but also to the effects of the expectations of its future actions. This point is best summarised by the quote from Woodford (1999a):

> *"One of the most important issues in the conduct of monetary policy, that should attain particular significance in an era of price stability, is the need to take account of the effects of the central bank's conduct upon private-sector expectations."*, Woodford (1999a) p. 1

2.2 The central bank

In this model it is assumed that society delegates monetary policy to an independent central bank. The central bank is required to minimise the discounted future expected value of a loss function assigned by society. The central bank's behaviour is therefore best described by the following objective function

$$\underset{x_t}{min}\, E_t(1-\beta)\sum_{i=0}^{\infty}\beta^i L_{t+i} \tag{2.5}$$

The central bank sets the value of i_t which minimises the discounted future expected value of the loss function, L_{t+i}. However, the formulation of the relation between i_t and x_t in the aggregate demand relation in (2.2) allows for a simpler assumption, which is that the central bank controls the output gap perfectly through the nominal interest rate. Hence, it will suffice to find the value of x_t that solves (2.5). The term, L_{t+i}, describes the loss function of the central bank.

Throughout the paper it is assumed that society is concerned with deviations in both inflation from the social optimal level of inflation and deviations in the output gap. The following loss function obtains such preferences

$$L_{t+i} = \frac{1}{2}\left(\lambda x_{t+i}^2 + \pi_{t+i}^2\right) \tag{2.6}$$

Chapter 6 in Woodford (2003) shows that the minimisation problem in (2.6) is in accordance with the representative household's utility function defined over consumption and leisure.[5]

The quadratic notation on both the output gap and inflation means that society is equally concerned with positive and negative deviations in the two variables. As mentioned above it is assumed that society is concerned with deviations in inflation from the social optimal level of inflation. For simplicity, the social optimal level of inflation is normalised to zero. The parameter, λ, determines society's preference for a stable output gap relative to stable inflation. The approximation in chapter 6 in Woodford (2003) finds an explicit relationship for λ depending on the structural parameters of the model. In this analysis it is, however, convenient to assume that λ equals a fixed true value. Society's preferences can be justified in the following manner: households are concerned with deviations in the output gap since it will imply inefficient consumption smoothing, while deviations in inflation are costly to firms because sticky prices will cause them to set inefficiently low or high prices.

It is assumed that central bank reappointment depends on the performance of monetary policy relative to the objective of society and furthermore, that society evaluates the central bank's average performance. It is therefore convenient to calculate the unconditional

[5]Formally, chapter 6 in Woodford (2003) shows that (2.6) evaluated in (2.5) results from a second-order Taylor approximation of the households utility function around steady state. Hence, monetary policy should aim at minimising the expected future discounted deviations in the output gap and inflation.

expected value of equation (2.6). Evaluating (2.6) in (2.5) this corresponds to

$$E\left(L_t\right) = var\left(x_t\right) + \lambda var\left(\pi_t\right) \tag{2.7}$$

for $\beta \to 1$.

2.2.1 The delegation process

The central bank is assumed to operate in a targeting regime. Following the common definition of a targeting regime in chapter 8 of Walsh (2010), a targeting regime is defined by the variables in the loss function, L_{t+i}, assigned to the central bank by society and the relative weight, λ, put on these variables.

The loss function in (2.6) characterises one targeting regime which is often referred to as *inflation targeting*. It is assumed that society chooses the relevant targeting regime. Hence, even though society's preferences correspond to inflation targeting, monetary policy may be based on a different targeting regime. This can be in terms of the relevant variables included in the loss function or the relative preference for the variables. The attractiveness of this delegation process will be clear later in the analysis. Society sets the relevant targeting regime by assigning a loss function to the central bank that corresponds to the desired targeting regime.

For society to be able to assign a loss function which is different from its own an additional assumption is needed. If the central bank is assumed to share society's preferences then assigning a different loss function to the central bank would require the central bank to set a policy which differs from its own beliefs. One way to go about this is to assume that the central bank enters into a contract with society, as analysed in Walsh (1995), which assures that the central bank has the right incentives to set monetary policy according to the desired targeting regime. This analysis will, however, be based on the simpler assumption that society can credibly appoint a central bank with preferences that corresponds to the desired targeting regime. Hence, if society wishes to change the targeting regime and thereby assign the central bank a new loss function then it would actually have to appoint a new central bank with the corresponding preferences.[6] This implies that the monetary policy makers may have different preferences than society which may be a more restrictive assumption. Nonetheless, it follows Rogoff (1985).[7]

No matter how the central bank's loss function is formed, equation (2.7) is used to evaluate

[6] To further clarify the validity of this assumption, one may view the corresponding problem related to fiscal policy. If society wishes to change the fiscal policy from a socialist to a liberal regime then it would also need delegate fiscal policy to a liberal government as one would not expect a socialist government to credibly carry out a liberal policy.

[7] In Rogoff (1985) society is able to delegate monetary policy to a central bank with a stronger preference for stable inflation than society.

the performance of the respective policy. Hence, the performance of monetary policy depends, throughout the analysis, on its ability to minimise the variance in the output gap and inflation, even though the output gap and inflation may not be included in the central bank's loss function.

3 History dependent monetary policy

When the private sector is forward-looking the central bank is not only able to affect the current state of the economy by its current actions, but also by influencing expectations regarding its future actions. This section will clarify this point by looking at the solution to monetary policy when the central bank operates in a regime of inflation targeting. This policy has become increasingly relevant since an increasing number of central banks have adopted some form of policy explicitly focusing on minimising the deviations in inflation from an inflation target during the previous two decades.[8]

In this analysis inflation targeting is defined as a targeting regime characterised by the loss function stated in equation (2.6).[9] Hence, the central bank is assumed to set monetary policy according to the following objective function

$$\underset{x_t}{min}\,\frac{1}{2}E_t\left(1-\beta\right)\sum_{i=0}^{\infty}\beta^i\left(\lambda x_{t+i}^2+\pi_{t+i}^2\right) \tag{3.1}$$

The social optimal level of inflation then denotes the central bank's inflation target.

Two solutions to monetary policy under inflation targeting are of interest for the analysis and are therefore the focal point of this section. Kydland and Prescott (1977) showed that the central bank reaches a better outcome for monetary policy if it is able to make some form of commitment to future monetary policy. By committing to future policy actions the central bank is able to influence the private sector's expectations about the future development of inflation and output. There are many ways the central bank can commit, however, in Clarida et al. (1999) *the optimal commitment solution* to monetary policy is derived. It implies that the central bank makes a fully unconstrained commitment to future monetary policy.[10] The central bank is, however, commonly assumed to be unable to commit, which effectively means the optimal commitment solution may not have much practical relevance. The solution, nonetheless, provides valuable information on the characteristics of optimal monetary policy and the optimal utilisation of the private sector's expectations.

Because it is common to assume that the central bank is unable to make any credible promise about future monetary policy, the first part of this section is devoted to the

[8] In 1990 New Zealand was the first country to adopt an inflation target. Since then an increasing number of central banks have adopted target for inflation. Schmidt-Hebbel (2009) lists 28 inflation targeting central banks as of 2008.

[9] Svensson (1999a) refers to the policy as *flexible* inflation targeting, since monetary policy is delegated to a central bank that is concerned with deviations in both the output gap and inflation. The special case where $\lambda = 0$ is then referred to as *strict* inflation targeting because monetary policy is then delegated to a central bank which is only concerned with deviations in inflation.

[10] Another way for the central bank to commit is to make a constrained commitment by, for example, committing to an instrument rule such as the *Taylor rule* described in Taylor (1993).

solution to inflation targeting when the central bank is forced to act discretionary. As Clarida et al. (1999) note, a central bank that sets monetary policy discretionary in a regime of inflation targeting fits best with reality.[11] Hence, this solution serves as a good benchmark for the analysis in this thesis. Any policy recommendations should at least be able to better utilise the private sector's expectations and improve on the outcome resulting from *the discretionary solution* to inflation targeting. The last part of this section reviews alternative targeting regimes which attempts on this when the central bank is forced to act under discretion.

The solution to monetary policy is a result of the behaviour of the central bank given by (3.1) conditioned on the structural equations describing the economy introduced in section 2.1. The problem is formally solved in two stages. Since x_t is assumed to be treated as the control variable, the central bank first chooses the optimal value of x_t and the endogenous state variable, π_t, conditional on the exogenous state variable, u_t. This amounts to solving the minimisation problem in equation (3.1) conditional on the Phillips curve in equation (2.1). Second, the central bank sets the value of its instrument, i_t, that results in the desired values of x_t and π_t. This amounts to solving the aggregate demand equation (2.2) for i_t. As mentioned in section 2.2 it is assumed that the central bank perfectly controls x_t. The following analysis therefore concentrates on solving the first stage of the policy problem and the aggregate demand relation (2.2) is therefore ignored when deriving the solution to monetary policy.

The formal solution method borrows from Clarida et al. (1999) and Vestin (2006). Although this section is concerned with the outcome for monetary policy under a policy focused on inflation, it is convenient for later comparisons to express the solutions in terms of the price level. As will be clear later, the optimal choices of x_t and p_t evolves according to

$$p_t = \theta_1 p_{t-1} + \theta_2 u_t$$

$$x_t = -\psi_1 p_{t-1} - \psi_2 u_t$$

The solution to monetary policy then amounts to determining the value of the coefficients θ_1, θ_2, ψ_1 and ψ_2.

3.1 Discretionary solution to inflation targeting

When the central bank is unable to commit it has to take the private sector's expectations about the future as given. The central bank then has to determine the outcome for monetary policy discretionary by setting the optimal policy period-by-period. Formally,

[11] Since the inability to commit continues to be an obstacle for central banks the argument in Clarida et al. (1999) remains valid.

this amounts to solving the following optimisation problem

$$\min_{x_t} \frac{1}{2}\left(\lambda x_t^2 + \pi_t^2\right) + \hat{F}_t \tag{3.2}$$

subject to

$$\pi_t = \kappa x_t + \hat{f}_t \tag{3.3}$$

taking the future expectations characterised by $\hat{F}_t \equiv \frac{1}{2} E_t \left[\sum_{i=1}^{\infty} \beta^i \left(\lambda x_{t+i}^2 + \pi_{t+i}^2\right)\right]_t$ and $\hat{f}_t \equiv \beta E_t \pi_{t+1} + u_t$ as given. The details of the derivation given in appendix 8.2 show that this leads to the following optimal trade off between inflation and the output gap

$$x_t = -\frac{\kappa}{\lambda}\pi_t \tag{3.4}$$

Equation (3.4) implies a *leaning against the wind* policy. The central bank should contract demand below the potential level whenever inflation rises above the target, and expand demand above the potential level whenever inflation falls below target. This reaction is stronger if prices are more sticky and weaker if the central bank has stronger preferences for output gap stability. Equation (3.4) is used to find the following solutions to x_t and p_t

$$p_t = p_{t-1} + \hat{\theta}_2 u_t \tag{3.5}$$

$$x_t = -\hat{\psi}_2 u_t \tag{3.6}$$

where

$$\hat{\theta}_1 = 1$$

$$\hat{\theta}_2 = \frac{\lambda}{\kappa^2 + \lambda\left(1 - \beta\rho\right)}$$

$$\hat{\psi}_1 = 0$$

$$\hat{\psi}_2 = \frac{\kappa}{\lambda}\hat{\theta}_2 = \frac{\kappa}{\kappa^2 + \lambda\left(1 - \beta\rho\right)}$$

and $\hat{\theta}_2, \hat{\psi}_2 > 0$. To evaluate the performance of the discretionary solution to inflation targeting against the alternative policies that are considered later using equation (2.7), the variance of x_t and π_t has to be calculated. Note first, however, that (3.5) is easily rewritten on the form $\pi_t = \hat{\theta}_2 u_t$ to express the outcome for inflation. The variances of inflation and the output gap are then given by

$$var(\pi_t) = \hat{\theta}_2^2 \sigma_u^2 \tag{3.7}$$

$$var(x_t) = \hat{\psi}_2^2 \sigma_u^2 \tag{3.8}$$

where $\sigma_u^2 = \frac{1}{1-\rho^2}\sigma_e^2$ follows from the definition of the cost-push shock from section 2.1.

3.2 The optimal commitment solution to inflation targeting

Clarida et al. (1999) and Woodford (1999b) show that the optimal commitment solution to inflation targeting is a result of the central bank making a fully unconstrained commitment. An unconstrained commitment implies that the central bank is able to commit to a future path for monetary policy which depends, not only on current shocks to the economy, but on the entire future path of shocks to the economy. The potential problems with commitment policies and the possible solutions to these problems are discussed later. For this reason, there will be made no explicit assumptions now about how the central bank is able to convince the public of the credibility of its unconstrained commitment. Hence, it is simply assumed that it is capable of doing so.

Given the previous assumptions made about monetary policy this form of commitment will have the central bank behave according to the objective function in equation (3.1), while committing, at time t, to a state contingent sequence for x_{t+i} for $i = 0, 1, 2, ...$ Following the solution method in Clarida et al. (1999), the following Lagrangian is defined

$$\min_{\{x_i\}_{i=t}^{\infty}} \frac{1}{2} E_t \left\{ \sum_{i=0}^{\infty} \beta^i \left[\lambda x_{t+i}^2 + \pi_{t+i}^2 + 2\phi_{t+i} \left(\pi_{t+i} - \beta \pi_{t+i+1} - \kappa x_{t+i} - u_{t+i} \right) \right] \right\} \tag{3.9}$$

where $2\phi_{t+i}$ is the multiplier on the Phillips curve.

Solving equation (3.9) leads to the following first-order conditions

$$\phi_{t+i} = \frac{\lambda}{\kappa} x_{t+i},\ i \geq 0 \tag{3.10}$$

$$\pi_{t+i} = -\left(\phi_{t+i} - \phi_{t+i-1} \right),\ i \geq 1 \tag{3.11}$$

$$\pi_t = -\phi_t \tag{3.12}$$

Combining the first-order conditions then leads to the following trade off for monetary policy

$$\pi_{t+i} = -\frac{\lambda}{\kappa} \left(x_{t+i} - x_{t+i-1} \right), \quad \forall i \geq 1 \tag{3.13}$$

$$\pi_t = -\frac{\lambda}{\kappa} x_t \tag{3.14}$$

with details of the derivation available in appendix 8.3. The optimal commitment policy also implies a leaning against the wind policy. In the initial period, when the policy is implemented, the central bank should react as if it were unable to commit and change the level of the output gap in response to deviations in inflation. However, in all subsequent periods the central bank should adjust the change in the output gap.[12] The latter characterises the central bank's optimal utilisation of the private sector's forward-looking

[12] Svensson and Woodford (2005) considers a variant of the optimal commitment trade off, which is the so-called *inflation forecast targeting*. Under this policy, the central bank faces the expectational version of

expectations. Recall, that section 2.1 showed how current variables essentially depends on the private sector's expectations about the future.

The reduced form expressions of the price level and the output gap are then given by

$$p_t = \tilde{\theta}_1 p_{t-1} + \tilde{\theta}_2 u_t \tag{3.15}$$

$$x_t = -\tilde{\psi}_1 p_{t-1} - \tilde{\psi}_2 u_t \tag{3.16}$$

where

$$\tilde{\theta}_1 = \frac{\left(\lambda\left(1+\beta\right)+\kappa^2\right)\left(1-\sqrt{1-4\beta\left(\frac{\lambda}{\lambda(1+\beta)+\kappa^2}\right)^2}\right)}{2\lambda\beta}$$

$$\tilde{\theta}_2 = \frac{\tilde{\theta}_1}{1-\beta\tilde{\theta}_1\rho}$$

$$\tilde{\psi}_1 = \frac{\left(1-\tilde{\theta}_1\beta\right)\left(1-\tilde{\theta}_1\right)}{\kappa}$$

$$\tilde{\psi}_2 = \frac{1-\tilde{\theta}_2\left[1+\beta\left(1-\rho-\tilde{\theta}_1\right)\right]}{\kappa}$$

and $0 < \tilde{\theta}_1 < 1$ and $\tilde{\theta}_2, \tilde{\psi}_1, \tilde{\psi}_2 > 0$. Note, that (3.15) can be rewritten to express inflation under the optimal commitment. It is optimal for the central bank to set inflation according to $\pi_t = -(1-\tilde{\theta}_1)p_{t-1} + \tilde{\theta}_2 u_t$. Thus, the optimal commitment policy implies that current inflation and the current output gap depends on the lagged state variable, p_{t-1}, further clarifying how this policy depends on the past. The variance of inflation and the output gap under the optimal commitment is then calculated

$$var(\pi_t) = \tilde{\xi}_1^2 \sigma_u^2 \tag{3.17}$$

$$var(x_t) = \tilde{\xi}_2^2 \sigma_u^2 \tag{3.18}$$

where the general solutions to ξ_1^2 and ξ_2^2 which are reused later are

$$\xi_1^2 = \frac{2\theta_2^2\left(1-\rho\right)}{\left(1-\theta_1\rho\right)\left(1+\theta_1\right)}$$

$$\xi_2^2 = \frac{\theta_2^2\psi_1^2\left(1+\theta_1\rho\right)+\psi_2^2\left(1-\theta_1^2\right)\left(1-\theta_1\rho\right)+2\rho\theta_2\psi_1\psi_2\left(1-\theta_1^2\right)}{\left(1-\theta_1^2\right)\left(1-\theta_1\rho\right)}$$

The derivation of ξ_1^2 and ξ_2^2 is provided in appendix 8.4.

the commitment trade off given by $E_t\pi_{t+1} = -\frac{\lambda}{\kappa}\left(E_t x_{t+1} - E_{t-1}x_t\right), \quad \forall i \geq 1$ and $E_t\pi_{t+1} = -\frac{\lambda}{\kappa}E_t x_{t+1}$ respectively. Hence, this policy requires the central bank to focus on the evolution of the forecastable components of inflation and the output gap.

3.2.1 History dependent vs. purely forward-looking monetary policy

Before moving on to discussing the differences between the two policies analysed above it is convenient to show that the optimal commitment to inflation targeting actually improves the trade off between inflation and output gap variance. As mentioned earlier society has the option of delegating monetary policy to a central bank with different relative preferences for stable inflation and output. It is therefore reasonable to evaluate the implied trade off in inflation and output gap variance for the two policies for different values of λ. Note from (3.7), (3.8), (3.17) and (3.18), that the variance of inflation and the output gap under both policies depends on λ.

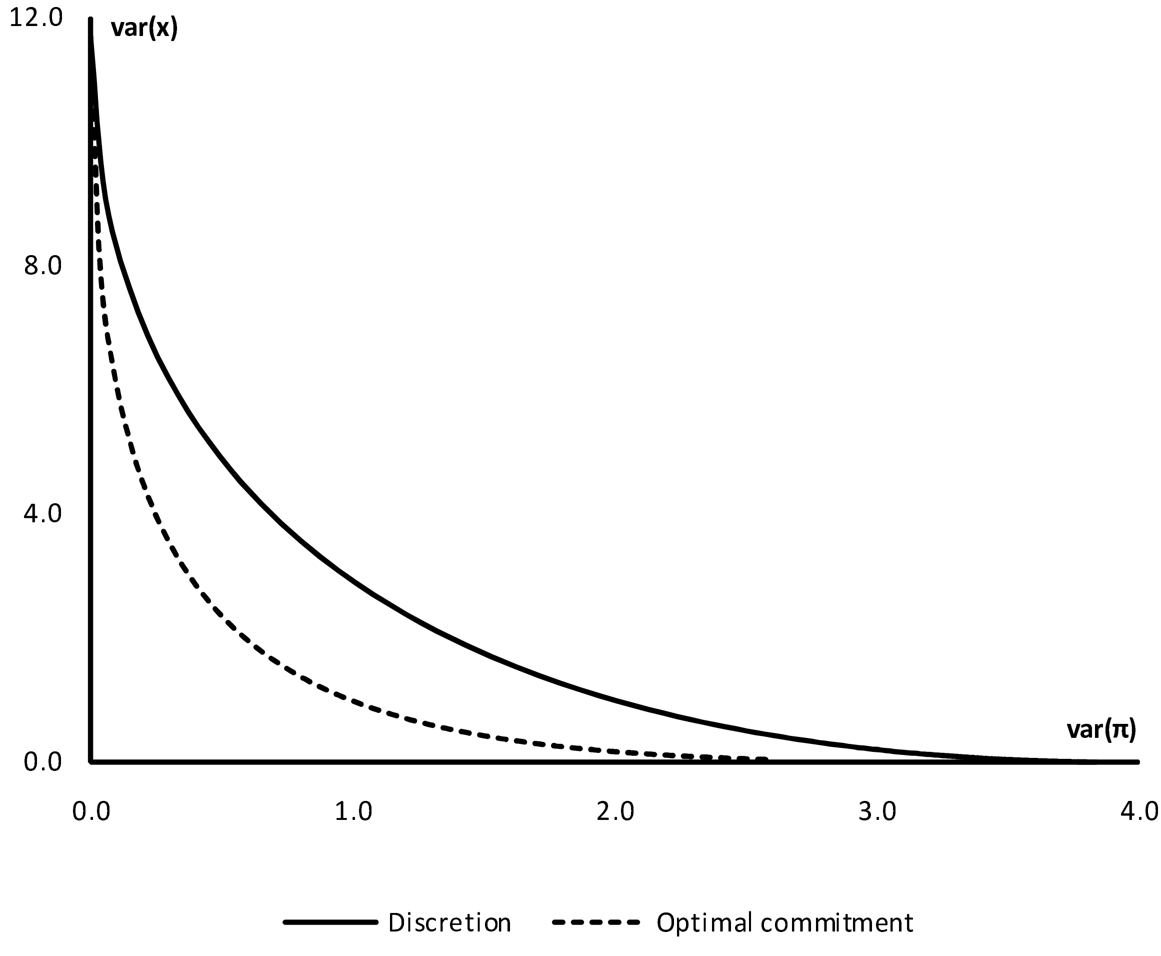

Figure 3.1: Policy frontier

To compute the policy frontier for the discretionary and the optimal commitment solution the parameter values, $\kappa = \frac{1}{3}$ and $\rho = \frac{1}{2}$, following the examples in Vestin (2006) are used. The computation is done by keeping κ and ρ fixed and calculating pairs of inflation and output gap variances for different values of λ. The policy frontier is illustrated in figure 3.1. The further the policy frontier is located to the north-east the greater is the implied welfare loss according to equation (2.7). Two things should be noted from the figure. One, the two policy frontiers do not intersect. Two, the policy frontier for the discretionary regime is located further to the north-east corner. Hence, the former point, that society always gains if it delegates monetary policy to a central bank that is able to commit, is confirmed.

To illustrate the difference between the two policies an impulse response to a cost-push shock is simulated. Figure 3.2 illustrates the reaction in inflation, the output gap and the price level to a temporary cost-push shock of one percent that lasts one period and then vanishes completely. The shock is assumed to hit in the subsequent period following the delegation of monetary policy. Society's relative preference for output gap stabilisation is assumed to be $\lambda = \frac{1}{4}$.

The top panel of figure 3.2 depicts the impulse response of the cost-push shock on the price level. When the central bank is unable to commit the cost-push shock persists and increases the price level permanently. However, when the central bank is able to make the optimal commitment it corrects the increase in the price level and the price level moves back towards its initial level. This difference is visible from the solution to θ_1 under the two policies. The coefficient on the lagged price level in the solution to the price level under discretion in (3.5) is $\hat{\theta}_1 = 1$, which imparts a unit root in the price level. On the other hand, under the optimal commitment policy given by (3.15) the coefficient is $0 < \tilde{\theta}_1 < 1$, which implies a stationary price level.[13] Hence, the optimal commitment to monetary policy implicitly involves committing to a future path for the price level. Following the discussion in Barnett and Engineer (2000), the optimal commitment policy may just as well be labelled a *price level* targeting policy as it implies an implicit price level target. This will be evident from the analysis in section 4.

This different response under the two policies results in the so-called *stabilisation bias*. When the central bank sets monetary policy discretionary the solutions to inflation and the output gap in (3.5) and (3.6) can be characterised as *purely forward-looking*, since they do not depend on any lagged variables. This in turn implies that the entire adjustment following the cost-push shock takes place immediately. In the period after the shock has hit inflation equals the target and the output gap is closed. This impulse response is illustrated in the middle and bottom panel of figure 3.2. When the central bank is able to make the optimal commitment the solutions to inflation and the output gap given by

[13] Appendix 8.6 shows that $0 < \tilde{\theta}_1 < 1$ holds for all values of $\lambda \in (0, \infty)$. Hence, the stationarity property of the price level is only violated if society appoints a central bank which is either concerned only about inflation stability or output stability. The explanation for this is rather intuitive. If the central bank only cares about inflation stability then $\tilde{\theta}_1 = 0$ and shocks to the price level is expected to be reverted immediately. If the central bank only cares about output stability then $\tilde{\theta}_1 = 1$ and shocks to the price level persists. This is because the central bank has no need to optimally utilise the private sector's inflation expectations as it is only concerned about output which it is able to fully control through the aggregate demand relation.

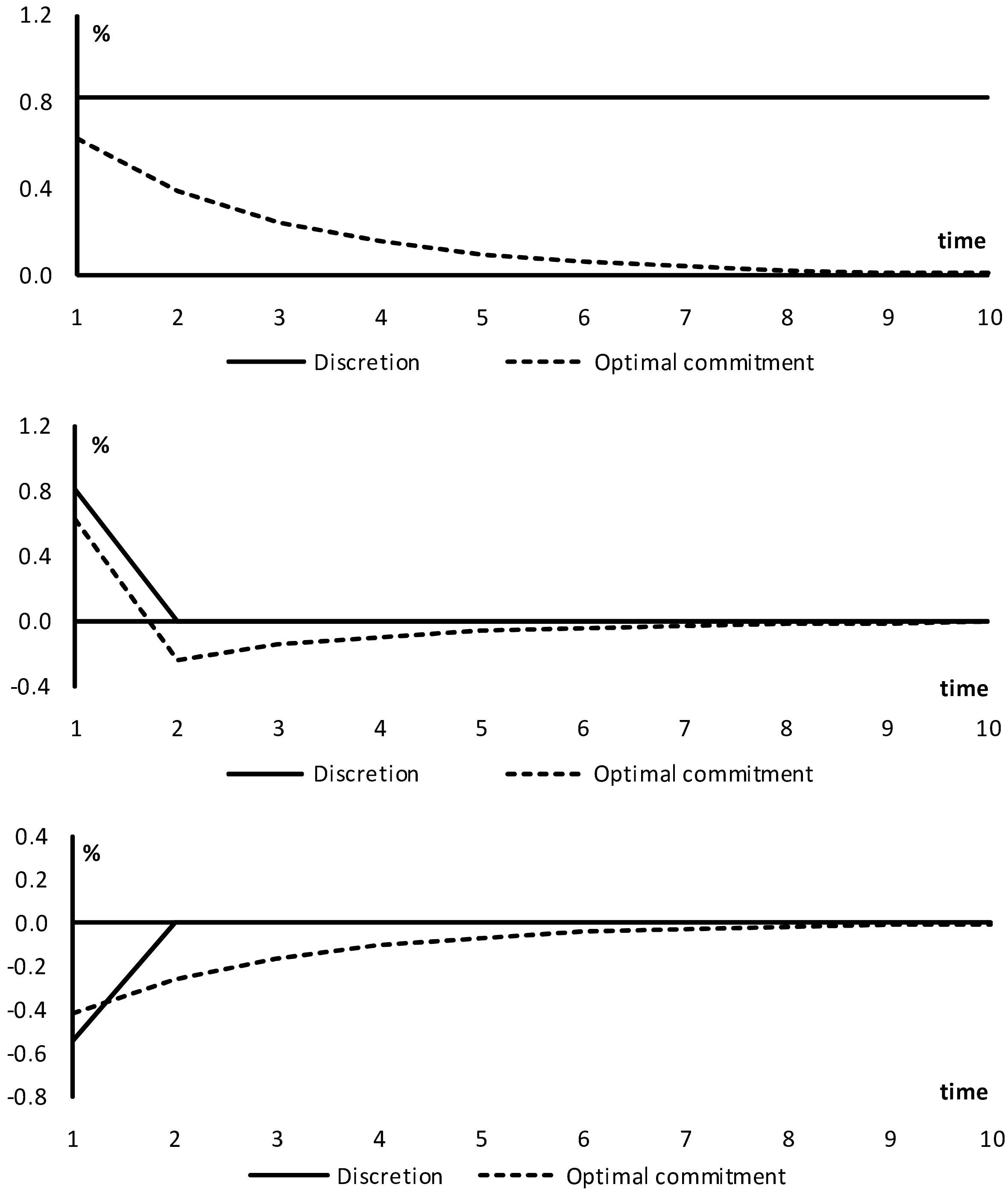

Figure 3.2: Impulse response on price level (top), inflation (middle) and output gap (bottom)

(3.15) and (3.16) can be characterised as *history dependent* because they include the lagged price level. As the bottom panel of figure 3.2 shows the central bank, in this case, commits to keep contracting demand which keeps inflation below the target as the middle panel of figure 3.2 illustrates. Because the commitment is assumed to be made credibly the private sector will correctly anticipate the reaction from the central bank and adjust expectations accordingly. Hence, expectations serve as an additional stabiliser. Consequently, the impulse response to inflation and the output gap is lower in the period the shock hits compared to the case where the central bank cannot commit. The stabilisation bias thus arises because of the inability of the central bank to optimally utilise the private sector's expectations when it is unable to make the optimal commitment. Conversely, a central bank which is able to make the optimal commitment is letting the market do some of stabilisation. Woodford (2000) surveys the distinction between purely forward-looking and history dependent policies.

Summing up, in an economy characterised by a forward-looking private sector optimal monetary policy is characterised by history dependence and a stationary price level.

3.2.2 Time inconsistency

As mentioned in the introductory notes to this section, research on monetary policy often assume that the central bank is not able to make a credible commitment to the future. This is because of the overall problem facing the optimal commitment policy, namely that it is not time consistent. A time consistent commitment implies that the policy planned by the central bank in period t for period $t+i$ remains the optimal policy once period $t+i$ arrives. Under the optimal commitment solution, however, there is a difference between the *ex ante* optimal policy and the *ex post* optimal policy. The first-order conditions implied by the optimal commitment given by equations (3.13) and (3.14) will help clarify this point. In period t it is optimal to commit to a future path for monetary policy, however, already when period $t + 1$ arrives it is optimal to abandon the commitment made in period t and instead reoptimise.

Figure 3.2 further illustrates this point. Under the optimal commitment policy the central bank does not have to contract demand as much in period t because it is able to optimally utilise the private sector's expectations about future monetary policy. However, once the temporary shock vanishes in period $t + 1$ it is instead optimal for the central bank to abandon the initial optimal commitment and reoptimise. In period $t + 1$ it is optimal for the central bank to set the policy implied by the discretionary solution. This leads to zero inflation, closes the output gap and a resulting gain in welfare. However, if the private sector realises that it is optimal for the central bank to abandon the optimal commitment it will adjust its expectations about future monetary policy accordingly. Consequently, the central bank will not be able to reap the gains of the optimal commitment in period t because the private sector will not perceive the commitment to be credible.

The success of the optimal commitment policy therefore depends on the institutional setup of monetary policy. A strong institutional setup helps assure the credibility needed for the private sector not to doubt the optimal commitment. There are different ways of building strong institutions. One approach is to concentrate on the central bank's incentives for not abandoning the optimal commitment. Abandoning the optimal commitment one time will result in a better outcome for monetary policy. However, it comes at the expense of the loss of credibility and the inability to make the optimal commitment in the future. If the central bank is punished for not keeping its promise then that could result in the proper incentives for not abandoning the optimal commitment.[14] Under this approach the central bank is given full operational flexibility to set monetary policy.

Another approach concentrates on constraining the central bank's operational flexibility. This could, for example, be done by specifying a rule that prescribes how the central bank is to set its monetary policy instrument in response to shocks to the economy. One example of such rule is the targeting rule implied by the aggregate demand relation in (2.2) and the solutions to the price level and the output gap und in (3.15) and (3.16) under the optimal commitment.[15] However, as described in Jensen (2011) an infinite number of instrument rules may also be considered.[16]

The benefit of a rule-based approach is that it increases the transparency of monetary policy. It enables the private sector to accurately predict the future path for monetary policy and any deviations from this path. In turn this helps make the optimal commitment credible. However, the approach does not remove the incentives of the central bank to deviate from the rule to improve the outcome for monetary policy. Combining the rule-based approach with the possibility of punishing the central bank for deviating from the rule may, however, improve on this.[17]

[14] A horrific example of this approach took place in March 2010 in North Korea when a senior economic official was executed following a failed currency reform. Although the threat of capital punishment probably will secure that the central bank has the right incentives it will most likely have the unwanted consequence of potential central bank candidates withdrawing their candidacy.

[15] Combining the three equations and solving for i_t leads to the following targeting rule, $i_t = \left(1 - \frac{\sigma\kappa}{\lambda}\right) E_t\pi_{t+1} + \sigma g_t$. Clarida et al. (1999) and Woodford (1999b) note, that this rule does not include determinacy properties because $1 - \frac{\sigma\kappa}{\lambda} < 1$ and it therefore does not satisfy the *Taylor principle* described in Taylor (1993), see footnote 16 on the Taylor principle. However, Jensen (2011) argues that the targeting rule will lead to determinacy because it is the rule resulting from optimising behaviour by the central bank.

[16] The instrument rule is not the result of optimising behaviour by the central bank. A necessary requirement for the instrument rule to secure determinacy is that it satisfies the Taylor principle. The Taylor principle described in Taylor (1993) prescribes that the central bank should adjust the interest rate more than one-to-one to changes in inflation.

[17] However, as noted in McCallum (1995) this actually just relocates the temptation to deviate from the central bank to the legislature responsible for appointing the central bank and punishing it.

3.2.3 The arbitrary t_0

A credible commitment to the targeting rule derived in footnote 15 would mean that the central bank is able to obtain the first-best solution to monetary policy given by the optimal commitment policy. Hence, before completely disregarding the practical relevance of the optimal commitment policy it is apparent to present one way to make the commitment easier.

The argument against the rule-based approach is that it is often considered a once-and-for-all commitment to a specific policy. This commitment in turn depends on what was desirable at the particular point in time when the rule was specified. Consequently, the central bank may not be able to respond optimally to unforeseen shocks which were not considered a possibility when the rule was adopted. Furthermore, it leaves little room for future improvement when ensuing research has added to the understanding of how the economy functions. Svensson (1999a) debates these arguments in further detail.[18]

This argument applies to both constrained and unconstrained commitment policies. However, it may turn out to be more hurtful if the central bank makes a once-and-for-all optimal commitment. This is because the history dependent characteristics of the policy, meaning that commitment not only depends on what was desirable at the point of commitment, but also on the state of the economy at the particular point in time.

As the former part of the analysis found, the optimal commitment policy implies that the central bank commits to a future path for the price level. This path depends on the price level in the period where the optimal commitment was made. Iterating equation (3.15) backwards will clarify this point.

$$p_t = \tilde{\theta}_1^{t-t_0+1} p_0 + \tilde{\theta}_2 \sum_{i=0}^{t-t_0} \tilde{\theta}_1^i u_{t-i} \tag{3.19}$$

Equation (3.19) shows that the current price level depends on the initial price level, p_0, when the optimal commitment was made and the history of shocks. The optimal commitment policy is often referred to as a policy which *does not let bygones be bygones*. The latter part of (3.19) justifies this label. Hence, the optimal commitment depends on the arbitrariness of the economic state at time t_0.

Woodford (1999a) proposes a way for the central bank to make the optimal commitment without having to rely on the arbitrary t_0 and thus in turn make the commitment easier.

[18]Further arguments in Svensson (1999a) are of a more practical matter. If an instrument rule is considered, which rule should be adopted? Even if a simple instrument rule such as the Taylor rule is considered. What parameter values should be chosen? The outcome for monetary policy can prove highly sensitive to this decision. Furthermore, a rule would leave monetary policy adjustments to become highly mechanical, which a computer could just as well handle. This point could contribute to explain any resistance from central banks towards a rule-based approach.

What is proposed in Woodford (1999a) is that the central bank instead makes the optimal commitment from *a timeless perspective.* A verbal definition of the timeless perspective is found in Woodford (1999a):

> *"The way that this can be done is for the central bank to adopt, not the pattern of behaviour from now on that it now would be optimal to choose, taking previous expectations as given, but rather the pattern of behaviour to which it would have wished to commit itself to at a date far in the past, contingent upon the random events that have occurred in the meantime."* Woodford (1999a) p. 18

If the central bank makes the optimal commitment from a timeless perspective it will not have to be concerned with the arbitrary t_0. This form of commitment instead allows the central bank to readjust the policy setup in any period if needed. Thus, making the optimal commitment from a timeless perspective is a way of making the commitment simpler. It does not, however, make the commitment time consistent. There is still a need for strong institutions in order for the central bank not to abandon its commitment.

To derive *the optimal timeless commitment solution* the first-order conditions of the time dependent optimal commitment solution stated in equations (3.10), (3.11) and (3.12) are reconsidered. When the central bank makes the optimal commitment from a timeless perspective it ignores (3.12), which means that the policy implied by the optimal timeless commitment is given by equation (3.13) only. The central bank then commits to following this policy in *all* periods. The price level under the optimal timeless commitment policy is found by letting t_0 approach minus infinity in equation (3.19) which gives

$$p_t = \tilde{\theta}_2 \sum_{i=0}^{\infty} \tilde{\theta}_1^i u_{t-i}$$

Blake (2001) and Jensen and McCallum (2002) find a policy which dominates the optimal timeless commitment. They consider the solution to the undiscounted version of (3.1) from a timeless perspective. They find that the policy which solves this problem is given by

$$\pi_t = -\frac{\lambda}{\kappa}\left(x_t - \beta x_{t-1}\right) \tag{3.20}$$

The solution is not easily derived analytically. However, both Blake (2001) and Jensen and McCallum (2002) confirm its optimality using numerical comparisons of this policy and the two alternatives in (3.4) and (3.13). (3.20) is labelled *the optimal fully timeless commitment.*

If the central bank makes an optimal fully timeless commitment it is able to improve the outcome of monetary policy. McCallum (2005) and Woodford (2010), however, note that the practical difference between the central bank making a timeless commitment and a fully timeless commitment, hence the difference between (3.13) and (3.20), may not be highly significant. When $\beta = 1$ the two policies lead to the same outcome. In reality the discount factor is close to one for quarterly data and the two policies are therefore quantitatively similar on a quarterly basis. Furthermore, commitment to the inflation targeting objective from a fully timeless perspective does not make the commitment time consistent.

3.3 Replicating the optimal commitment

Since there is no obvious ways of making the optimal commitment fully credible it is apparent to look at other ways of improving institutions which does not rely on the central bank's ability to commit. A new approach, which has gained a lot of interest in the literature, considers the possibility of implementing the optimal commitment policy by assigning an alternative targeting regime to the central bank. The idea is that the central bank is allowed to act under discretion as in section 3.1, but it is assigned a different loss function which alters its objective and leads to a better utilisation of the private sector's forward-looking expectations which in turn imparts history dependence to the solution to monetary policy. Hence, the point of this approach is to concentrate on what objective the central bank announces for monetary policy in order to influence what the private sector expects the central bank to do in the future. Recall, that section 2.2 discussed the necessary assumptions needed for this form of delegation of monetary policy to be credible.[19]

The literature has found a family of different targeting regimes which impart history dependence to monetary policy, thus, a better utilisation of the private sector's expectations compared to the discretionary solution to inflation targeting. The following part of this section reviews the most popular of the alternatives. One important note must be made about the literature review. The reviewed targeting regimes all induce history dependent behaviour into monetary policy. However, this way of improving the institutional setup does not necessarily lead to a solution where monetary policy is equivalent to the optimal commitment policy. Furthermore, a history dependent policy does not necessarily improve the monetary policy trade off compared to inflation targeting. One example of the latter point is found in Yetman (2003). Here a history dependent policy which does worse than inflation targeting is analysed.[20]

[19] As mentioned in footnote 17, McCallum (1995) argues that this delegation approach simply relocates the time inconsistency problem from the central bank to legislature which appoints the central bank. However, McCallum (1995) also argues that commitment may not be an obstacle for the central bank.

[20] The example in Yetman (2003) is a modified inflation targeting regime where the central bank is required to correct for past deviations from the inflation target.

3.3.1 Review of the literature

The literature presents a number of targeting regimes which impart history dependence in the solution to monetary policy and thus replicates the general characteristic of the optimal commitment policy. They are all a special case of the general version of the quadratic loss function given by

$$L_{t+i}^{V}=\frac{1}{2}\left[w_x x_{t+i}^2+w_\pi \pi_{t+i}^2+\sum_{n=1}^{m} w_v^n\left(v_{t+i}^n\right)^2\right] \tag{3.21}$$

It is easily seen that equation (3.21) incorporates inflation targeting as a special case, when $w_x=\lambda$, $w_\pi=1$ and $w_v^n=0$ for $n=1,...,m$.

Walsh (2003) analyses the so-called *speed limit policy*. The speed limit policy requires society to delegate monetary policy to a central bank with preferences described by the following setting $w_x=0$, $w_\pi=1$, $v_t^1=x_t-x_{t-1}$, $w_v^1=\lambda$ and $w_v^n=0$ for $n=2,...,m$ in (3.21). That corresponds to the following loss function

$$L_{t+i}^{SL}=\frac{1}{2}\left[\pi_{t+i}^2+\lambda\left(x_{t+i}-x_{t+i-1}\right)^2\right]$$

The central bank is then concerned with deviations in inflation and furthermore seeks to smooth changes in the output gap.[21] As the results from Walsh (2003) show the inclusion of the lagged output gap makes the solution to monetary policy history dependent.

Another example which is related to the speed limit policy is analysed in Jensen (2002). Jensen (2002) looks at the outcome for monetary policy, when society delegates monetary policy to a central bank that is concerned with deviations in nominal income growth. Under this policy, labelled *nominal income growth targeting*, the third term in (3.21) takes the form $v_t^1=\Delta n_t\equiv\pi_t+(y_t-y_{t-1})$, $w_v^1>0$ and $w_v^n=0$ for $n=2,...,m$. The parameters, w_x and w_π, can take on different values depending on what form of nominal income growth targeting is considered. The analysis in Jensen (2002) includes flexible nominal income growth targeting where $w_x=\lambda$, $w_\pi=0$ and the corresponding loss function is

$$L_{t+i}^{NIGT}=\frac{1}{2}\left[\lambda x_{t+i}^2+w_v^1\Delta n_{t+i}^2\right]$$

Strict nominal income growth targeting corresponds to $w_x=0$. Walsh (2003) considers a third form of nominal income growth targeting, where monetary policy is delegated to a central bank that is concerned with nominal income growth and inflation. Hence, $w_x=0$ and $w_\pi=1$ under this policy. Finally, Jensen (2002) also considers the outcome of the combination regime, when the central bank has preferences for the output gap,

[21] Remember that the output gap is defined as $x_t\equiv y_t-y_t^f$. The preference for output gap smoothing, x_t-x_{t-1}, is thus equivalent to $(y_t-y_{t-1})-(y_t^f-y_{t-1}^f)$. Hence, the speed limit policy requires the central bank to minimise the deviations in actual output growth from potential output growth.

inflation and nominal income growth. The parameter setting is then adjusted accordingly so $w_x = \lambda$ and $w_\pi > 0$. No matter how the nominal income growth targeting regime is designed the desire of the central bank to smooth nominal income growth implicitly adds the lagged value of output to the loss function. This property leads to a solution to monetary policy which is history dependent.

Delegating monetary policy to a central bank which is concerned with lagged values of inflation has also been shown to produce a history dependent policy. Nessén and Vestin (2005) analyse the implications for monetary policy when society appoints a central bank with an *average inflation* preference. Following (3.21), this policy is best described by $w_x = \bar{\lambda}$, $w_\pi = 0$, $w_v^1 = 1$, $v_t^1 = \bar{\pi}_{j,t}$, where $\pi_{j,t} \equiv \frac{1}{j}\sum_{s=0}^{j-1}\pi_{t-s}$ and $w_v^n = 0$ for $n = 2, ..., m$ and thus the following loss function

$$L_{t+i}^{AIT} = \frac{1}{2}\left[\bar{\lambda}x_{t+i}^2 + \pi_{j,t+i}^2\right]$$

As with standard inflation targeting, the special case where $\omega_x = 0$ corresponds to strict average inflation targeting. j determines over how many periods the targeted average inflation is calculated. $j = 1$ corresponds to the standard inflation targeting case analysed in section 3.1, where $j = 2, 3, 4..$, corresponds to average inflation targeting. A central bank with an average inflation preference will respond to high inflation in the past by aiming at low inflation in the future. A larger j will have monetary policy depend on a longer history.

This point is further emphasised by considering an average inflation target calculated over a large number of periods. This will roughly correspond to *price level targeting* as analysed in Vestin (2006) where the central bank's loss function takes the form

$$L_{t+i}^{PT} = \frac{1}{2}\left[\bar{\lambda}x_{t+i}^2 + (p_{t+i} - \bar{p})^2\right]$$

and $\bar{p}$ is the price level target. Under price level targeting the central bank aims at reverting any deviations in the price level from the target $\bar{p}$, which will impart history dependence into the solution to monetary policy.

Batini and Yates (2003), Cecchetti and Kim (2005) and Røisland (2006) consider a *hybrid inflation/price level targeting* loss function where instead $v_t^1 = p_t - \varrho p_{t-1}$ while the remaining setting equals the setting under average inflation targeting. Hence, the loss function takes the form

$$L_{t+i}^{HIPT} = \frac{1}{2}\left[\bar{\lambda}x_{t+i}^2 + (p_{t+i} - \varrho p_{t+i-1})^2\right]$$

When $0 \leq \varrho < 1$ the central bank puts some weight on a price level target and some weight on an inflation target. The special case of $\varrho = 0$ corresponds to the price level targeting regime described above and $\varrho = 1$ corresponds to standard inflation targeting.[22] Hence, $0 \leq \varrho < 1$ imparts history dependence to the solution to monetary policy.

Woodford (1999b) considers the case where monetary policy is delegated to a central bank that cares about *interest rate smoothing*. The loss function is then modified to take on the following setting $w_x > 0$, $\omega_\pi = 1$, $v_t^1 = i$, $w_v^1 > 0$ and $w_v^n = 0$ for $n = 2, ..., m$ to give

$$L_{t+i}^{IRS} = \frac{1}{2}\left[w_x x_{t+i}^2 + w_\pi \pi_{t+i}^2 + w_v^1 \left(i_{t+i}\right)^2\right]$$

This policy leads to inertial interest rate setting by the central bank and the solution to monetary policy becomes history dependent.[23]

3.3.2 Comparing the regimes

The former review presented a number of alternative targeting regimes that enables a history dependent solution to monetary policy without the central bank having to make a binding commitment to future actions. However, the different alternatives do not yield the same trade off for monetary policy. As the proceeding analysis in section 4 will show price level targeting can perfectly replicate the optimal commitment solution. Furthermore, the study in Walsh (2003) shows that the monetary policy trade off under price level targeting is superior to both the speed limit policy and various forms of nominal income growth targeting. Nessèn and Vestin (2005) find that price level targeting yields a better trade off than average inflation targeting.

However, these results are highly dependent on the assumptions made about the formation of expectations in the economy. Recall, that section 2.1 showed how current inflation and the current output gap are functions of current and expected future variables. A more general version of the model than the one used here allows for a fraction of the private sector to make decisions based on a backward-looking rule-of-thumb. Modelling this behaviour could, for example, be done by including the lagged value of inflation in equation (2.1) and the lagged output gap in equation (2.2), hence, allowing for endogenous persistence in the model.

Of the different targeting regimes considered above price level targeting is generally found superior to the other alternatives when decisions are based entirely on expectations about the future. The different model calibrations presented in Jensen (2002), Walsh (2003) and Nessèn and Vestin (2005), however, reveal that the relative conclusions change in favour

[22] The label hybrid inflation/price level target is further clarified by the alternative formulation $v_t^1 = (1-\varrho)p_t + \varrho(p_t - p_{t-1}) = (1-\varrho)p_t + \varrho\pi_t$, whereby it is clear that ϱ denotes the weight put on the inflation target.

[23] Yetman (2004) shows that the speed limit policy and the price level targeting policy may just as well be characterised as interest rate smoothing policies. Yetman (2004) proves this by substituting an appropriate formulated aggregate demand relation into the loss function.

of the other alternatives when a fraction of the private sector makes decisions based on a backward-looking rule-of-thumb.[24] Which policy is preferable when not all private agents are forward-looking depends on how large the fraction of backward-looking private agents is. They all, however, find that if a large fraction of the private sector is backward-looking then the superiority tilts in favour of inflation targeting.

Walsh (2003) finds price level targeting to be preferable over the speed limit policy, nominal income growth targeting[25] and inflation targeting when no more than approximately a third of the private sector is backward-looking. When instead the fraction of backward-looking agents is between one third and two thirds the conclusion changes in favour of the speed limit policy with nominal income growth targeting as the second best alternative. Finally, when more than two thirds make decisions based on a rule-of-thumb inflation targeting is the preferred alternative.

Furthermore, Nessèn and Vestin (2005) find that when approximately half of the private sector is backward-looking average inflation targeting outperforms price level targeting.[26] When a large fraction of firms are backward-looking they find that standard inflation targeting outperforms price level targeting and average inflation targeting. In the remaining case price level targeting is the superior policy.

Røisland (2006) finds that if a fraction of firms set prices based on a backward-looking rule-of-thumb then it is optimal for the central bank to let the weight on the inflation target in the hybrid inflation/price level targeting regime equal that fraction.[27] Hence, Røisland (2006) thus finds that when all firms are backward-looking then optimal monetary policy is inflation targeting.

[24] Jensen (2002) and Walsh (2003) include lagged inflation in the Phillips curve and the lagged output gap in aggregate demand relation. Nessèn and Vestin (2005) include both lagged inflation and the lagged output gap in the Phillips curve.

[25] Walsh (2003) considers what Jensen (2002) refers to as flexible nominal income growth targeting in his comparison.

[26] Nessèn and Vestin (2005) prove this for an average inflation target calculated over 6, 9 and 12 periods respectively.

[27] Røisland (2006) does not include any endogenous persistence in the aggregate demand relation.

4 Price level targeting

Of the different alternatives presented in section 3.3 price level targeting stands out as an especially interesting option for monetary policy. The literature review found price level targeting to be the optimal policy in the model introduced in section 2. Furthermore, the optimal commitment solution suggests that optimal monetary policy should be concerned with the price level.

The next part of the analysis will therefore investigate the outcome, when society appoints a central bank with preferences for price level stability. The derivations borrow from Vestin (2006).[28]

It is assumed that the central bank operates in the form of price level targeting described in section 3.3. Hence, the targeting regime is described by the following loss function

$$L_{t+i} = \frac{1}{2}\left[\bar{\lambda}x_{t+i}^2 + p_{t+i}^2\right] \tag{4.1}$$

where the price level target is zero.[29] A central bank with preferences given by (4.1) will not let shocks to the price level persist.[30] Furthermore, it should be noted that society not only appoints a central bank with a preference for the price level. It appoints a central bank with a relative preference for a stable output gap given by $\bar{\lambda}$, which may be different from society's relative preference for a stable output gap given by λ.[31] Note finally, that although society appoints a central bank with a different objective for monetary policy the central bank's performance is still evaluated according to (2.7) describing the true preferences of society.

It is convenient for the following analysis to restate the Phillips curve in equation (2.1) in terms of the price level using $\pi_t \equiv p_t - p_{t-1}$

$$p_t - p_{t-1} = \beta\left(E_t p_{t+1} - p_t\right) + \kappa x_t + u_t \tag{4.2}$$

[28] Although the positive results on price level targeting in this thesis builds on Vestin (2006), it should, rightfully, be mentioned that the potential superioty of price level targeting over inflation targeting was first highlighted by the so-called "free lunch" in Svensson (1999b). The analysis in Svensson (1999b) finds price level targeting to be superior in a Neoclassical model when output displays some persistence. The result was highly debated in related articles. Kiley (1998) doubts the robustness of the result to the choice of model, Ditmar et al. (1999), on the other hand, confirm the results in Svensson (1999b) for the case where the central bank cannot commit and Dittmar and Gavin (2000) find support for price level targeting in a New Keynesian model.

[29] Yetman (2005) compares inflation targeting and price level targeting when the central bank's inflation target takes a positive value. A positive inflation target corresponds to a price level target with a drift. Hence, the central bank targets a path for the price level which grows at the rate of the inflation target.

[30] The limitations to this argument presented in footnote 13 also holds under price level targeting which is also showed in appendix 8.6. Except under price level targeting it is the central bank's preference for output stability relative to its preference for price level stability that matter.

[31] Note, however, the important distinction between $\bar{\lambda}$ and λ. $\bar{\lambda}$ describes the preference for output gap stability relative to price level stability, while λ describes the preference for output gap stability relative to inflation stability.

Note from equation (4.2), that the lagged price level now enters as an additional state variable in the central bank's optimisation problem.

4.1 Solution to price level targeting

Because the appointed central bank is unable to commit to the future path for monetary policy it will, as in section 3.1, take the private sector's expectations about the future as given when it sets monetary policy. Hence, it reoptimises period-by-period. The formal problem of the central bank is then to solve

$$\min_{x_t} \frac{1}{2}\left(\bar{\lambda}x_t^2 + p_t^2\right) + \bar{F}_t \tag{4.3}$$

subject to

$$p_t - p_{t-1} = -\beta p_t + \kappa x_t + \bar{f}_t \tag{4.4}$$

taking the expectations about the future given by, $\bar{F}_t \equiv \frac{1}{2}E_t\left[\sum_{i=1}^{\infty}\beta^i\left(\bar{\lambda}x_{t+i}^2 + p_{t+i}^2\right)\right]_t$ and $\bar{f}_t \equiv \beta E_t p_{t+1} + u_t$, as given. Appendix 8.5 adds details on the derivation of the above optimisation problem which leads to the following optimal relations for p_t and x_t

$$p_t = \bar{\theta}_1 p_{t-1} + \bar{\theta}_2 u_t \tag{4.5}$$

$$x_t = -\bar{\psi}_1 p_{t-1} - \bar{\psi}_2 u_t \tag{4.6}$$

where

$$\bar{\theta}_1 = \frac{\omega\bar{\lambda}}{\kappa^2 + \omega^2\bar{\lambda} + \beta\bar{\lambda}\left(1 - \omega\bar{\theta}_1\right)}$$

$$\bar{\theta}_2 = \frac{\omega\bar{\lambda} + \beta\rho\bar{\lambda}\left[2\omega\bar{\theta}_2 - \left(1 + \beta\rho\bar{\theta}_2\right)\right]}{\kappa^2 + \omega^2\bar{\lambda} + \beta\bar{\lambda}\left(1 - \omega\bar{\theta}_1\right)}$$

$$\bar{\psi}_1 = \frac{\left(1 - \bar{\theta}_1\beta\right)\left(1 - \bar{\theta}_1\right)}{\kappa}$$

$$\bar{\psi}_2 = \frac{1 - \bar{\theta}_2\left[1 + \beta\left(1 - \rho - \bar{\theta}_1\right)\right]}{\kappa}$$

and $\omega = 1 + \beta\left(1 - \bar{\theta}_1\right)$ and the coefficients are defined in the intervals $0 < \bar{\theta}_1 < 1$ and $\bar{\theta}_2, \bar{\psi}_1, \bar{\psi}_2 > 0$. Equation (4.5) can be rewritten on the form $\pi_t = -\left(1 - \bar{\theta}_1\right)p_{t-1} + \bar{\theta}_2 u_t$ so the variance of inflation and the output gap can be calculated from

$$var(\pi_t) = \bar{\xi}_1^2\sigma_u^2 \tag{4.7}$$

$$var(x_t) = \bar{\xi}_2^2\sigma_u^2 \tag{4.8}$$

where $\bar{\xi}_1^2$and $\bar{\xi}_2^2$ follow from the general expression of the coefficients stated in section 3.2.

The solution shares the properties of the optimal commitment solution found in section 3.2. Price level targeting also imparts history dependence and a stationary price level to monetary policy. Hence, following a positive cost-push shock the impulse response follows the response under the optimal commitment illustrated in figure 3.2. The central bank contracts demand to create deflation which will assure that the price level reverts back to the target. The private sector anticipates this response and adjusts expectations accordingly which limits the effect from the cost-push shock on current inflation compared to discretionary inflation targeting.

4.2 Replicating the optimal commitment solution

Even though price level targeting shares the same stabilising properties as the optimal commitment policy it does not, however, imply that the two policies are quantitatively equivalent. To determine whether this is the case the coefficients, $\bar{\theta}_1$ and $\bar{\theta}_2$, need to be examined.

The outcome of the price level determines the outcome of the output gap. To find out if price level targeting is quantitatively equivalent to the optimal commitment it is therefore sufficient to compare the outcome for the price level under the two policies. Looking at the outcome for the price level under the optimal commitment and price level targeting given by equations (3.15) and (4.5) it is easily seen that a sufficient condition for the two outcomes to equal is that

$$\tilde{\theta}_1 = \bar{\theta}_1$$

$$\tilde{\theta}_2 = \bar{\theta}_2$$

It is not, however, easily seen out of the expressions of $\tilde{\theta}_1$, $\bar{\theta}_1$, $\tilde{\theta}_2$ and $\bar{\theta}_2$ when this is satisfied.

4.2.1 No persistence in u_t

If there is no persistence in the cost-push shock process the problem, however, simplifies. From the expressions of $\tilde{\theta}_1$, $\bar{\theta}_1$, $\tilde{\theta}_2$ and $\bar{\theta}_2$ it is seen that if $\rho = 0$ then $\tilde{\theta}_1 = \tilde{\theta}_2$ and $\bar{\theta}_1 = \bar{\theta}_2$. Furthermore, $\tilde{\theta}_1$ is a function of λ, while $\bar{\theta}_1$ is a function of $\bar{\lambda}$. Recall from section 2.2, that it is assumed that society is able to set $\bar{\lambda}$ when appointing the central bank. Thus, with no persistence in the cost-push shock, price level targeting can replicate the optimal commitment if there exists a $\bar{\lambda}$ such that $\tilde{\theta}_1(\lambda) = \bar{\theta}_1(\bar{\lambda})$.

Appendix 8.6 shows that the following holds for the coefficients $\tilde{\theta}_1(\lambda)$ and $\bar{\theta}_1(\bar{\lambda})$

$$\lim_{\lambda \to 0} \tilde{\theta}_1(\lambda) = 0$$

$$\lim_{\lambda \to \infty} \tilde{\theta}_1(\lambda) = 1$$

and

$$\lim_{\bar{\lambda} \to 0} \bar{\theta}_1(\bar{\lambda}) = 0$$

$$\lim_{\bar{\lambda} \to \infty} \bar{\theta}_1(\bar{\lambda}) = 1$$

which imply

$$0 \leq \tilde{\theta}_1(\lambda) < 1$$

$$0 \leq \bar{\theta}_1(\bar{\lambda}) < 1$$

Hence, both $\tilde{\theta}_1(\lambda)$ and $\bar{\theta}_1(\bar{\lambda})$ are defined over the interval $[0, 1)$. Furthermore, both $\tilde{\theta}_1(\lambda)$ and $\bar{\theta}_1(\bar{\lambda})$ are continuous functions of λ and $\bar{\lambda}$ respectively.[32] Remember, that society's relative preference for a stable output gap, λ is assumed to be fixed at the true value. Thus for a fixed value of $\tilde{\theta}_1(\lambda)$ it is therefore possible to find a value of $\bar{\lambda}$ which implies $\tilde{\theta}_1(\lambda) = \bar{\theta}_1(\bar{\lambda})$.

Hence, an important result emerges. When there is no persistence in the cost-push shock process, then it is always possible to perfectly replicate the optimal commitment solution. Under this condition price level targeting thus yields a first-best solution to monetary policy which does not require the central bank to make any commitments about the future.

4.2.2 Persistence in u_t

When there is persistence in the cost-push shock process the conditions

$$\tilde{\theta}_1(\lambda) = \bar{\theta}_1(\bar{\lambda})$$

$$\tilde{\theta}_2(\lambda) = \bar{\theta}_2(\bar{\lambda})$$

have to be satisfied. It turns out that there does not exist a $\bar{\lambda}$ that satisfies this condition. This is best illustrated by plotting the θ_1 and θ_2 coefficients under the optimal commitment policy and price level targeting for different values of λ. Figure 4.1 does this using the same values for κ and ρ as in section 3.2.

[32] It is trivial to see that this is satisfied for $\tilde{\theta}_1(\lambda)$. As Vestin (2006) also notes, it is possible to solve $\bar{\theta}_1(\bar{\lambda})$ explicitly for $\bar{\theta}_1$. Doing that would result in a third-order polynomial with one real continuous solution in the interval $[0, 1)$ which is the solution of interest here.

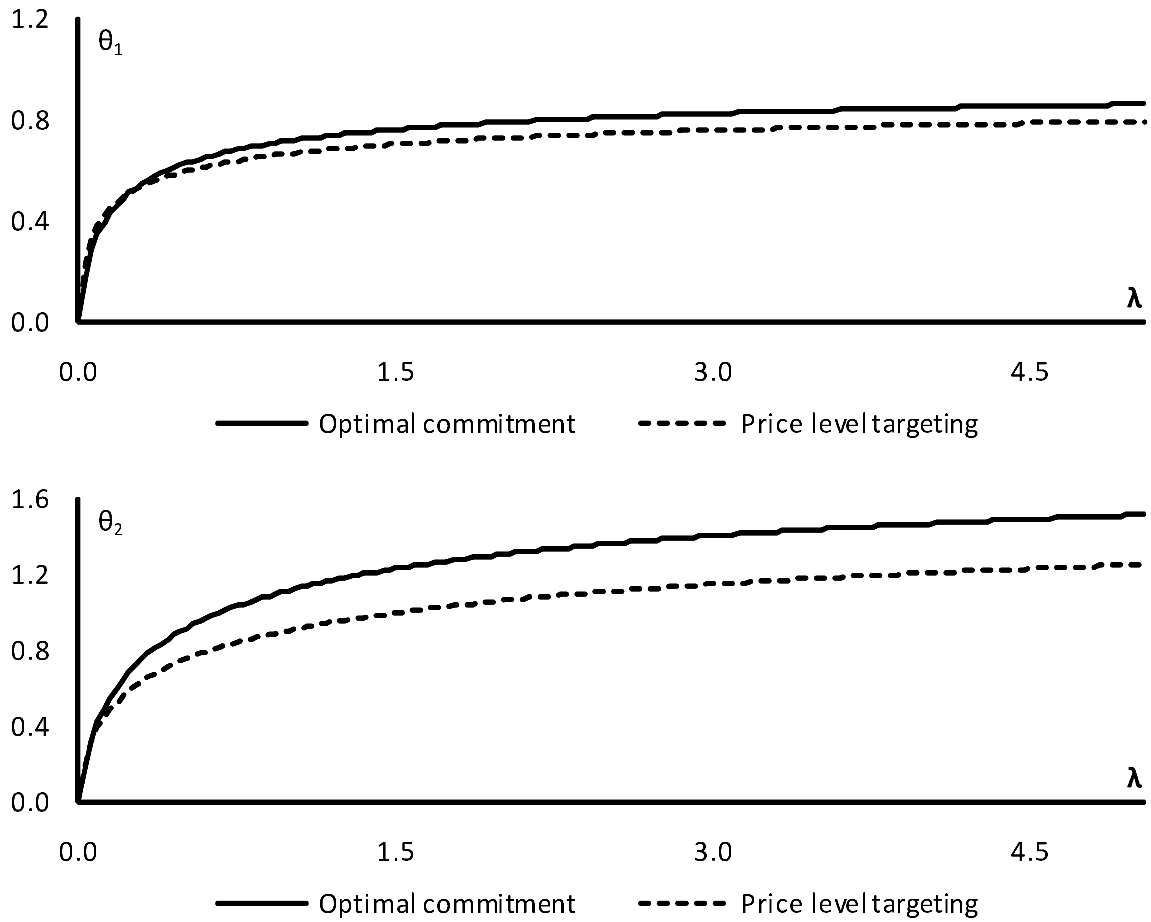

Figure 4.1: θ_1(top) and θ_2(bottom)

Hence, when there is persistence in the cost-push shock process it is not possible to perfectly replicate the optimal commitment. It is therefore necessary to prove that price level targeting, nonetheless, presents a second-best solution to monetary policy. Thus, it is apparent to show that price level targeting improves the inflation-output variance trade off compared to the discretionary solution to inflation targeting.

To show that price level targeting actually succeeds in this it will suffice to include the outcome for price level targeting in the policy frontier plot in figure 3.1 because society is able to set the value of $\bar{\lambda}$ when appointing the central bank. The policy frontier including the trade off under price level targeting given by (4.7) and (4.8) is plotted in figure 4.2.

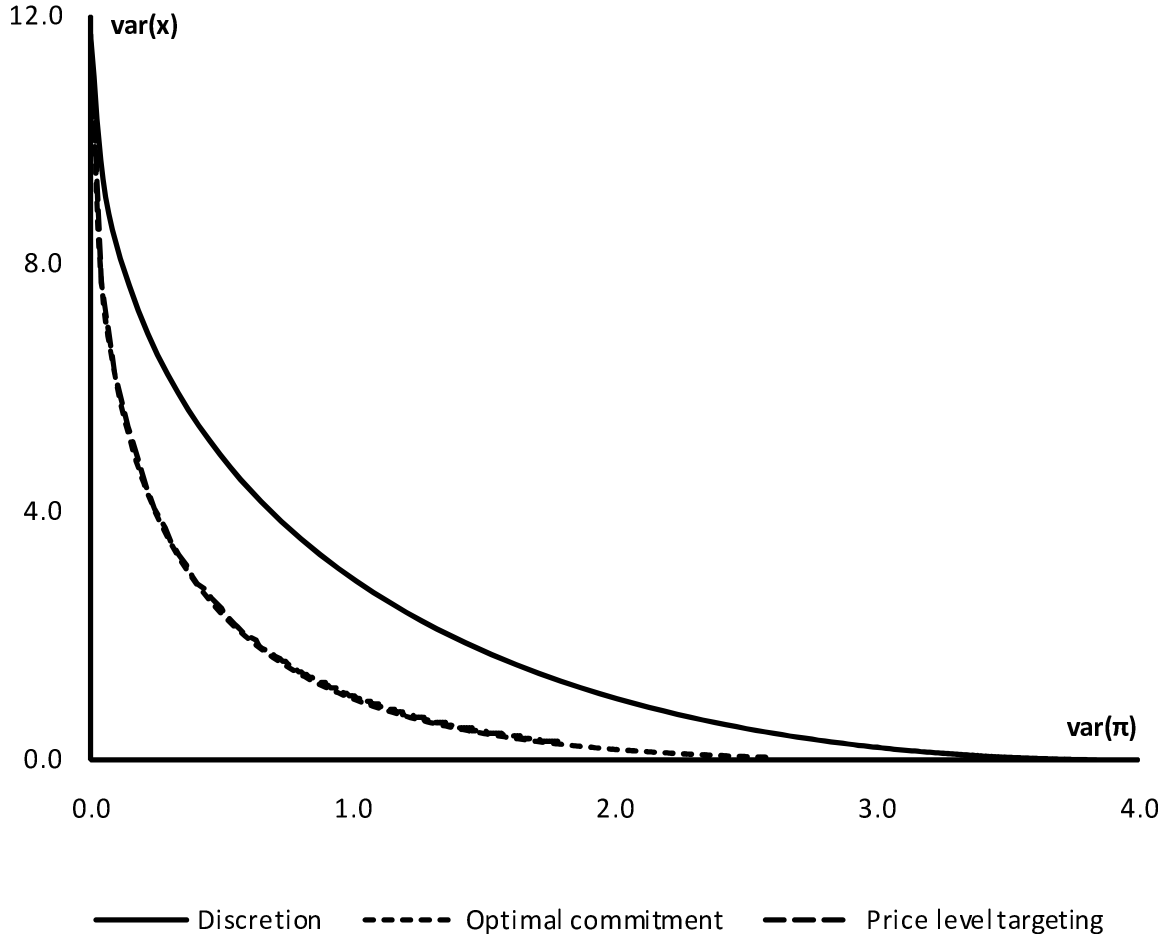

Figure 4.2: Policy frontier

Figure 4.2 shows that price level targeting leads to a better trade off between the variance of inflation and output compared to the discretionary solution to inflation targeting. It is furthermore worth noting, that even though price level targeting is not able to perfectly replicate the optimal commitment policy, the difference between the two policies looks to be minimal.

In summary: price level targeting presents an opportunity to improve monetary policy compared to inflation targeting when the central bank cannot make a credible commitment about future actions. Price level targeting improves monetary policy by replicating the characteristics of the optimal commitment solution implying a history dependent policy with a stationary price level. Furthermore, depending on the properties of the cost-push shock process it may be possible to perfectly replicate the optimal commitment policy with price level targeting.

4.3 The zero lower bound

Throughout the analysis it has been assumed that the central bank uses the short-term nominal interest rate as its monetary policy instrument without making any further assumptions about the limitations to this instrument. The implication of this assumption is, for example, that the central bank is able to fully offset all demand shocks by changing the nominal interest rate. However, because of the presence of money balances in the model a negative nominal interest rate will have no effect. This part of the analysis will therefore assume a zero lower constraint on the nominal interest rate. This amounts to restricting the nominal interest rate to all non-negative values, $i_t \geq 0$, in the model. The restriction, however, makes the solution to monetary policy non-linear which requires advanced mathematical tools to solve.[33] This analysis therefore concentrates on the qualitative impact on monetary policy of the non-negative constraint.

Analysing the problems for monetary policy when the zero lower bound on the nominal interest rate binds has great empirical relevance. The last decade monetary policy has been conducted in a low inflationary environment which has depressed inflation expectations, and consequently lowered nominal interest rates.[34] For this reason, the central banks have had less room to manoeuvre when negative demand shocks have hit the economy. A number of central banks have therefore been forced to lower the nominal interest rate to- or close to zero.[35]

In the context of price level targeting the endogenous role of inflation expectations plays a central role when the zero lower bound on the nominal interest rate binds. A large strain of literature have pointed out the attractiveness of the history dependent price level targeting compared to the purely forward-looking inflation targeting when the zero lower bound on the nominal interest rate binds. The main arguments and conclusions are repeated here. Gaspar and Smets (2000) use a numerical calibration of a version of the New Keynesian model similar to the one used in this paper to emphasise the attractiveness of price level targeting when the zero lower bound binds. The simulation shows that price level targeting results in less interest rate volatility which in turn implies that the problems with the zero lower bound become much less important compared to the case where the central bank has no concerns about price level stability. Barnett and

[33] The analysis presented in Eggertsson and Woodford (2003) and Wolman (2005) respectively finds a solution to the non-linear model by the use of advanced mathematical tools. See footnote 39 for more on the analysis in Eggertsson and Woodford (2003).

[34] Rogoff (2003) among others analyse this development. The analysis, for example, finds that annual inflation in the industrial economies dropped from a level of almost 9% in the early 1980s to around 2% in the early 2000s.

[35] The most recent examples of this has occurred following the outbreak of the Financial Crisis in 2008. Consequently, the Federal Reserve has set its policy rate close to zero since 2008, the Riksbank lowered the policy rate to 0.25% in 2009 and 2010 and Bank of England has set the policy rate at 0.50% since 2009. See also Svensson (2010), where it is argued that even though the policy rate is not lowered to zero it may, nonetheless, be constrained by the effective lower bound.

Engineer (2000) stress that a price level target is a good idea when the zero lower bound binds. This is because the strictly forward-looking expectations helps keep the interest rate operating procedure effective which limits the possibility of a liquidity trap. Wolman (2005) concludes that when the economy is characterised by staggered price-setting, then the zero lower constraint on the nominal interest rate has no real implication if monetary policy is delegated to a central bank which sets monetary policy according to a price level targeting objective. Mishkin (2006) finds that price level targeting is superior to inflation targeting in a deflationary environment. This is because the expected higher inflation induced by the price level target enables a lower real interest rate. This point is also available in the survey in Côté (2007). Côté (2007) finds that because the active expectational channel under price level targeting enables the real interest to continue to fall below zero when the nominal interest rate has been lowered to zero, then price level targeting may be better to protect against liquidity traps. Gaspar et al. (2007) conclude that the stabilising effect through expectations will both lead to less need for adjusting the nominal interest rate and furthermore keep monetary policy effective when the nominal interest rate is stuck at zero. Ambler (2009) concludes that monetary policy has more leverage near the zero lower bound under price level targeting compared to inflation targeting. This is because of the adverse response on inflation expectations following a negative shock to the price level. Cournède and Moccero (2009) show, using numerical simulations, that the risk of hitting the zero lower bound is greatly reduced under price level targeting compared to inflation targeting. Furthermore, the results also hold when only a fraction of firms are forward-looking in their price-setting. In Kahn (2009) it is concluded that a price level targeting policy improves monetary policy when the zero lower bound binds, while further reducing the risk of hitting the lower constraint. Schmidt-Hebbel (2009) concludes that because deflation is not bygones under price level targeting then the likelihood of ending up in a deflationary spiral is lower, and the likelihood of getting out of one is larger, under price level targeting.

To clarify the arguments made in the literature, the forward-looking version of the aggregate demand relation stated in equation (2.4) is a good place to start. It shows how demand essentially depends on the expected future real interest rates. Following *the Fisher equation,* the current *ex ante* real interest rate, r_t is

$$r_t = i_t - \pi_t^e$$

which is the nominal interest rate adjusted for current expectations about future inflation, π_t^e.

If the economy is in a recession caused by a shock to demand, hence, the current output gap is negative, then following (2.4), the expected future real interest rate must be too high. To raise the current output gap and help the economy out of recession the central bank can either try to impact the private sector's expectations about the future nominal interest rate and/or its expectations about future inflation.

Bernanke et al. (2004), Bernanke and Reinhart (2004) and Bullard (2010) among many others have analysed the first option. They suggest that the central bank can affect the expectations about the future nominal interest rate by using the so-called *extended period language*. This implies that the central bank promises to keep the nominal interest rate low either for a fixed period or conditional on the economic conditions. See Bank of Canada (2009) and Federal Reserve (2011b) for examples of such announcements. As it is argued in Svensson (2004) the gain from promising to keep the nominal interest rate low for an extended period may be low as it has probably already been lowered to- or close to zero. Another argument against this approach is that, unless nothing further is assumed, it involves a commitment by the central bank which is not time consistent. Consequently, one has to explain why the central bank should be able to commit to future monetary policy under certain economic circumstances.

Hence, the analysis will concentrate on the central bank's ability to impact the private sector's expectations about future inflation. Both inflation targeting and price level targeting enables the central bank to anchor the private sector's inflation expectations. The target thus serves as a nominal anchor. Without the presence of a nominal anchor the central bank may have to make use of unconventional methods of monetary policy to influence inflation expectations. This could, for example, involve expanding the monetary base.[36]

When monetary policy is delegated to a central bank with an inflation objective the inflation target anchors inflation expectations. This is easily seen from the inflation expectations under inflation targeting given by $E_t\pi_{t+1} = \bar{\pi}$, where $\bar{\pi}$ is the central bank's inflation target, included for the purpose of illustration, and no persistence in the cost-push shock has been assumed for simplicity. The expectations are found by iterating the solution to inflation under inflation targeting one period ahead and taking expectations. If society has fully understood and trusts that the central bank will revert any deviations in inflation from the target then it will have no reason not to expect that future inflation will equal the central bank's target. Under this form of policy the real interest rate is

$$r_t = i_t - \bar{\pi}$$

where inflation expectations are fixed at the fixed inflation target. It is evident that the

[36]In research of monetary policy alternatives when the zero lower bound binds this is often referred to as the so-called *quantitative easing*, see Orphanides and Wieland (2000), Bernanke et al. (2004), Bernanke and Reinhart (2004) and Bullard (2010) among others. Quantitative easing implies increasing the size of the central bank's balance sheet. For this to have an impact on inflation expectations the private sector has to perceive the increase to be permanent. However, this again raises the problem of commitment - see, for example, Svensson (2010) on this point.

central bank under this policy can affect the real interest rate until the nominal interest rate is lowered to zero. The inflation target will then assure that the real interest rate does not rise due to deflationary expectations.

When society appoints a central bank with a price level objective the price level target instead serves as an anchor for inflation expectations. This can easily be seen from the solution for inflation under price level targeting.[37] Under price level targeting inflation expectations are expressed as follows $E_t\pi_{t+1} = -(1-\bar{\theta}_1)(p_t - \bar{p})$ - derived in the same manner as above using the result to inflation under price level targeting. Hence, when the price level undershoots the target the private sector expects the central bank to create inflation in the future, so the price level reverts back to the target and *vice versa*. This form of policy thus implies a real interest rate of

$$r_t = i_t - \pi^e(p_t - \bar{p})$$

where the inflation expectations now is a function of the current deviation in the price level from target with the following property, $\frac{\partial \pi^e}{\partial (p_t - \bar{p})} < 0$. Hence, under price level targeting the central bank can still affect the real interest rate through the nominal interest rate. However, if a zero nominal interest rate is not sufficient to reach the monetary policy objective then the real interest rate will decrease further because of the endogenous inflation expectations. This is how price level targeting improves monetary policy compared to inflation targeting when the zero lower bound binds

Because the real interest rate is affected through two endogenous channels under price level targeting, while only through one under inflation targeting, the risk of hitting the zero lower bound is reduced. Price level targeting requires less variation in the nominal interest rate to adjust the real interest rate because the inflation expectations are doing part of the adjustment.

The importance for monetary policy of anchoring inflation expectations is one of the highly mentioned benefits of inflation targeting.[38] As the analysis above showed the inflation target enables the central bank to perfectly control the real interest rate. When the nominal interest rate is lowered to zero the inflation target, however, puts a lower constraint on the real interest rate, hence, $r_t \in [-\bar{\pi}, \infty)$. As Svensson (2010) notes, a real interest rate of, $r_t = -\bar{\pi}$, may not lead to sufficient demand stimulus though. A price level target, however, does not impose a lower constraint on the real interest rate, hence, $r_t \in (-\infty, \infty)$. When the nominal interest rate is lowered to zero the real interest rate instead is given by $r_t = -\pi^e(p_t - \bar{p})$ which depends endogenously on the price level. As

[37] The equivalent alternative to targeting a positive rate of inflation would be a price level target which growth at a rate corresponding to the inflation target. However, leaving this out will not affect the qualitative conclusions of the analysis.

[38] Walsh (2009a), for example, concludes that the ability of the central bank in responding to demand shocks may be increased by committing to an explicit inflation target.

long as the price level undershoots the price level target the private sector will continue to expect inflation in the future. This will in turn lower the real interest further.

To clarify this difference consider the effect under the two policies of a temporary negative demand shock which the central bank under both policies is assumed not to be able to fully offset by setting the nominal interest rate to zero. Thus, the nominal interest rate is set to zero in both cases. The economy is still assumed to be characterised by the model introduced earlier. Hence, in the case of inflation targeting the target is zero and under price level targeting the price level target is fixed. Under inflation targeting this leaves the real interest rate equal to zero. This is assumed not to create the necessary demand stimulus and the output gap is thus negative which results in deflation. Under price level targeting the demand shock also results in a negative output gap and deflation. However, because deflation causes the current price level to undershoot the price level target the private sector will increase its inflation expectations which lowers the real interest rate further and thus create additional demand stimulus.[39] The net outcome is a smaller loss in output and less deflation under price level targeting compared to inflation targeting.

To summarise, extending the analysis to a situation where the zero lower constraint on the nominal interest rate binds following a large demand shock, gives price level targeting additional leverage over inflation targeting. A relevant question is then whether a second-best option for an inflation targeting central bank is to remain with inflation targeting and only make a transition to price level targeting in the event the zero lower bound binds. One problem may be to define the price level target. To gain the advantages of price level targeting when the zero lower bound binds the central bank should aim for a price level which exceeds the current price level in order to increase inflation expectations. This may in turn require that the policy is adopted in a timeless perspective which enables the central bank to aim for the pre-recession price level. Walsh (2009b), however, doubts the credibility associated with adopting price level targeting in the midst of a recession. As section 4.5 shows, replacing inflation targeting with price level targeting may be associated with short-run costs as the private sector learns about the new regime.

4.4 Price level targeting with an escape clause

Price level targeting requires the central bank to aim at correcting any shocks to the actual price level. However, in given circumstances it may be optimal to adopt the policy

[39] In Eggertson and Woodford (2003) the model is numerically solved for the case of optimal commitment with the non-negative restriction on the nominal interest rate imposed on the aggregate demand relation. The optimal commitment implies that, if the central bank is unable to fully offset a demand shock by setting the nominal interest to zero, then it should set the nominal interest rate to zero. In the following period the central bank should then raise its price level target to correct for the previous period's shortfall. This will lead to a further increase in inflation expectations resulting in a lower real interest rate and additional demand stimulus.

response under inflation targeting and "forget about the shock" and reset the price level target. For such a policy response to be credible an explicit escape clause, allowing the central bank to reset the target conditional on special events, should be included.

Several inflation targeting regimes include an escape clause.[40] An escape clause augmented to an inflation targeting regime generally allows the central bank to suspend the target when inflation is affected by external factors the central bank has no control over.[41] Hence, in this case the escape clause limits the central bank to "target what it can hit".

Price level targeting does not treat bygones as bygones and it is therefore even more important for the central bank to "target what it can hit". If, for example, the price level index, which the central bank targets, moves above the target due to external factors, which the central bank has no control over, then the central bank is required to produce an unwanted contraction in demand. Including an escape clause in the price level targeting regime, which allows the central bank to ignore the shock and reset the target, will then enable the central bank not to react to the shock to the price level and instead treat it as bygones. A different solution to this problem would be to simply exclude the external factors from the relevant price level index.[42] In this case the central bank only "targets what it can hit" and there is no need for an escape clause. As Walsh (2009b) notes, if there is certain prices the central bank does not control then they should be excluded from the relevant price level index. This, however, remains a practical issue which will not be considered further.

Cost-push shocks present a trade off for monetary policy under both inflation targeting and price level targeting. While inflation targeting is only concerned with current cost-push shocks, price level targeting reacts to the entire history of cost-push shocks. If the economy is in a deep recession caused by a large cost-push shock then it may be optimal for a price level targeting central bank to ignore the shock and reset the target. Such

[40] The Czech National Bank and the Reserve Bank of South Africa explicitly list external supply shocks as a viable exemption of the inflation targeting policy. Norges Bank lists a number of factors which it is permitted not to take into account when it sets monetary policy. The appendix in Mishkin and Schmidt-Hebbel (2002) include more examples.

[41] An example of an escape clause is found in Czech National Bank (2004): *"The CNB continues to regard its inflation targets as medium-term targets from which actual inflation may deviate temporarily. Such a deviation comes into consideration especially if the economy is hit by an "exogenous shock". If such a shock deflects expected inflation from the target, the CNB does not respond to the primary impacts of the shock. It will apply an exemption ("escape clause") from the obligation to hit the inflation target and accept the deviation of the inflation forecast from the target caused in this way. There can be a whole range of shocks which create room for applying such escape clauses. They include, for example, major deviations in world prices of energy raw materials or major deviations in agricultural producer prices. A specific type of exogenous shocks is administrative measures that have strong price impacts, in particular major changes in the structure or rates of indirect taxes and major changes in the segment of regulated prices."*

[42] The central bank may consider targeting a consumer price index which excludes commodity prices determined on the global market and certain taxes and regulated prices controlled by the government. The former is of course only relevant in the case of an open economy. For a theoretical analysis of the relevant price level index for the central bank to target, see, for example, Mankiw and Reis (2003).

behaviour may be motivated by the experience of the gold standard under World War I. As with price level targeting, the gold standard does not let bygones be bygones. However, during World War I the policy was suspended and resumed when the war ended - for some countries at a different parity. Hence, even though the former analysis showed that price level targeting is superior to inflation targeting both when the economy is susceptible to demand shock and cost-push shocks this may, nevertheless, only be the case during normal business cycles. In turn this qualifies augmenting an escape clause to the price level targeting regime.

In all aspects, however, it is not of importance to the following analysis why an escape clause is present. It is the implications of including the escape clause to a price level targeting regime, which allows the central bank to reset the target under given circumstances, which is of interest.

4.4.1 Multiple equilibria

To formally analyse the implications of allowing a conditional reset of the price level target it is apparent to first formally justify the need for a target reset. This justification may be based on the latter argument made above. According to the society loss function in (2.6) society is equally concerned with positive deviations and negative deviations in the output gap. This may well be the true characterisation of society's preferences during normal business cycles. However, this part of the analysis assumes that society is not equally concerned with large positive and large negative deviations in the output gap. More specifically, it is assumed that the loss associated with a large *positive* cost-push shock, which pushes up inflation leading to a steep decline in demand, is greater for society than an equally large *negative* cost-push shock. Thus, a hostile invasion or a natural disaster is assumed to lead to a greater loss than the discovery of large unknown natural resources.[43]

The quadratic loss term on the output gap in society's loss function presented in equation (2.6) may therefore not describe well the cost of large declines in output. Borrowing from the analysis in Masson and Shukayev (2011), the following loss function presents one way of characterising such asymmetric preferences for large deviations in the output gap.

$$L_{t+i} = \frac{1}{2}\left[\lambda x_{t+i}^2 + \pi_{t+i}^2 + C(x_t)\right] \tag{4.9}$$

The loss function (4.9) adds the term $C(x_t)$ to (2.6). $C(x_t) = 0$ if $x_t > X$ and $C(x_t) > 0$ if $x_t \leq X$, where $X < 0$. It is furthermore implicitly assumed that the zero lower bound never binds. Hence, $C(x_t) > 0$ is the result of a large cost-push shock. This loss function

[43]Empirically, this assumption seems plausible if one, for example, considers the attention the oil price shocks in the early 1970s and 1980s during the so-called *first-* and *second oil crisis* has received. However, this may just be because large positive cost-push shocks have occurred more frequently than their negative counterparts.

induces an asymmetry to society's preferences due to deviations in the output gap. In the delegation process society will therefore want to consider appointing a central bank with equally asymmetric preferences for large deviations in the output gap.

Hence, the loss function of a central bank concerned with deviations in the price level and the equivalent asymmetric preferences for deviations in the output gap is given by

$$L_{t+i} = \frac{1}{2}\left[\bar{\lambda} x_{t+i}^2 + (p_{t+i} - \bar{p})^2 + C(x_t)\right] \tag{4.10}$$

where $C(x_t)$ is the only addition compared to the loss function under price level targeting in (4.1) and an explicit price level target has been added for the convenience of the following analysis.

It is then assumed that the central bank is able to mitigate the loss associated with a cost-push shock which leads to an output gap of $x_t \leq X$ by temporary letting bygones be bygones. Such behaviour corresponds to a price level targeting policy with an escape clause. It is not of interest to try and find an explicit solution for monetary policy when the central bank's preferences are given by (4.10). The loss function (4.10) is only used to formally motivate the inclusion of an escape clause.

To include an escape clause of the type described above the central bank is assumed to have a time varying price level target of the form

$$\bar{p}_t = \delta_t p_{t-1} + (1 - \delta_t)\bar{p}_{t-1}$$

where δ_t denotes the probability of resetting the target and letting bygones be bygones. The probability of a target reset can be viewed as a trigger strategy conditional on the decline in output under the price level targeting policy being large enough and conditional on the target reset resulting in a higher output. Hence, δ_t takes the form

$$\delta_t = \begin{cases} 1 \quad if \quad x_t^{\delta=0} \leq X \quad and \quad x_t^{\delta=0} < x_t^{\delta=1} \\ 0 \quad if \qquad\qquad\qquad\qquad otherwise \end{cases}$$

which in turn may be viewed as an escape clause.

Whether or not the central bank will reset the price level target depends on the current value of the output gap. The current value of the output gap is determined endogenously in the Phillips curve. The Phillips curve in equation (2.1) shows that the current output gap depends negatively on expected future inflation. Hence, if inflation expectations are affected by the inclusion of the escape clause then the possibility of multiple equilibria arises.

To derive an expression of the private sector's expectation about future inflation consider first the expression of current inflation given by

$$\pi_t = \delta_t \pi_t^{\delta=1} + (1 - \delta_t)\, \pi_t^{\delta=0}$$

which implies that inflation expectations are given by

$$E_t \pi_{t+1} = \Pr(\delta_{t+1} = 1)\, E_t \pi_{t+1}^{\delta=1} + (1 - \Pr(\delta_{t+1} = 1))\, E_t \pi_{t+1}^{\delta=0}$$

Hence, current and expected future inflation depends on the current and expected future conditional values of inflation.

The specific form of the escape clause implies that the conditional solution to inflation is on the same form as the solution under price level targeting if the target is not reset and on the form under inflation targeting if the target is reset. Staying consistent with the same notation as in section 4.1 the current and expected future values of conditional inflation are then given by

$$\pi_t^{\delta=0} = -\left(1 - \bar{\theta}_1\right)(p_{t-1} - \bar{p}_{t-1}) + \bar{\theta}_2 u_t$$

$$\pi_t^{\delta=1} = \bar{\theta}_2 u_t$$

$$E_t \pi_{t+1}^{\delta=0} = -\left(1 - \bar{\theta}_1\right)(p_t - \bar{p}_t) + \bar{\theta}_2 \rho u_t$$

$$E_t \pi_{t+1}^{\delta=1} = \bar{\theta}_2 \rho u_t$$

The private sector's expectation that the central bank will reset the target in period $t+1$ is formed in period t. To clarify this important point $\Pr(\delta_{t+1} = 1)$ is denoted ζ_t. Using this and the above expression then gives the following expression for inflation expectations

$$E_t \pi_{t+1} = -(1 - \zeta_t)\left(1 - \bar{\theta}_1\right)(p_t - \bar{p}_t) + \bar{\theta}_2 \rho u_t$$

which has the important property

$$\frac{\partial E_t \pi_{t+1}}{\partial \zeta_t} > 0$$

Hence, the private sector's expectations about future inflation depends positively on how great it values the probability that the central bank will reset the target in the future.

Consider then, the effect of a cost-push shock which pushes prices above the target, $p_t > \bar{p}_t$, and further results in a decline in output which is not large enough to trigger a target reset, $x_t > X$. When $p_t > \bar{p}_t$ the private sector will, *ceteris paribus*, expect deflation in the future. However, if the private sector values the likelihood of a future target reset as high then that limits the downward effect on inflation expectations. The reason for this is quite intuitive. If the central bank is expected to frequently reset the target the private sector will then expect the central bank to set monetary policy as if it had an inflation targeting objective and in turn monetary policy will not reap the gains of the optimal commitment policy. The consequence of this is a smaller effect from the deflationary expectations on the output gap which will result in a larger decline in output and a potential target reset. Hence, high expectations of a future target reset will eventually become self fulfilling.

The analysis in Masson and Shukayev (2011) further illustrates this point using numerical simulations. When the trigger is, $X = -8$, which corresponds to an output gap of -8%, then the simulations show that the unconditional probability of a price level target reset is 0.0%. However, when the trigger is, $X = -1$, corresponding to an output gap of -1%, then the reset probability increases to 12-15% depending on the level of credibility associated with monetary policy.[44]

Thus, adding an escape clause to the price level targeting policy, which includes the possibility of resetting the target, will have a negative effect on the stabilising effects working through the expectational channel. However, in the extreme case, where the central bank is expected to reset the target every period the private sector's expectations about future inflation will correspond to those under inflation targeting. Hence, the worst consequence of adding an escape clause to a price level targeting regime is that the gain from the policy compared to inflation targeting turns to zero.

4.5 Credibility of the target

The favourable conclusions about price level targeting hinges on the important assumption made in section 2.2 about the delegation process. It is assumed that society, with no loss of credibility, can assign a targeting regime that does not correspond to the preferences of society. The credibility issue of the assumption can overall be divided into two problems.

First, the assumption requires that there exists a central bank that has preferences for a price level target, or, as discussed in section 2.2, that it is possible to create a contract between society and the central bank which assures that the central bank aims at achieving the price level target. Limited empirical evidence actually supports the validity of this assumption. In the 1930s the Riksbank in Sweden implemented a price level targeting policy more-or-less equivalent to the targeting regime analysed in the former part of this section.[45] Furthermore, Bank of Canada has recently considered the possibility of

[44] Masson and Shukayev (2011) find at least two stable equilibria with different unconditional probability of a target reset. They denote the equilibrium with high unconditional probability of target reset a low credibility equilibrium and *vice versa* and run the simulations for the two equilibria. When the trigger is $X = -1$ they find an unconditional reset probability of 15.4% related to the low credibility equilibrium and a reset probability of 12.3% related to the high credibility equilibrium.

[45] The Swedish experience with price level targeting is analysed in Berg and Jonung (1999). The

replacing the inflation target with a price level targeting regime of the kind analysed here.[46] Hence, it is reasonable to assume that there exists a central bank with preferences for price level targeting.

Second, the assumption requires that the private sector is capable of understanding the implications of the price level targeting policy. As mentioned, inflation targeting has become a common targeting regime for monetary policy - see footnote 8. However, switching from inflation targeting to price level targeting would require the private sector to understand the implications of the arbitrary t_0 when the policy is implemented and that shocks to the price level are no longer ignored. Since, no central bank has made the transition from inflation targeting to price level targeting it is not possible to justify this assumption empirically. Furthermore, since the practical evidence with price level targeting is reduced to around eight years in Sweden in the 1930s it is highly difficult to make any empirical justification of whether it is reasonable to assume that the private sector is able to understand the implications of a price level targeting policy.

It is thus necessary to turn to the theoretical evidence on this matter. One way to analyse the problem would be to inspect the outcome under price level targeting if the private sector consistently misperceives the central bank's price level target. One could imagine that society has just replaced an inflation targeting central bank with a price level targeting central bank and that the private sector fails to understand the new target for the price level. Yetman (2003) looks at the outcome for monetary policy in this case and compares it to inflation targeting. The analysis assumes that the central bank sets monetary policy aiming at stabilising the price level around a target of $\bar{p}$, while the private sector instead believes that the central bank aims at a target of $\bar{p} + \varepsilon$, where $\varepsilon \neq 0$. The former analysis highlighted the importance of the price level target in the formation of expectations. If the central bank and the private sector have different beliefs about the price level target then they will also have different expectations about future inflation. Using the same model as here, the analysis in Yetman (2003) finds that if the private sector has different beliefs about the price level target then price level targeting will no longer be superior to inflation targeting. The analysis shows that this is true even for very small deviations in the perception about the price level target, while for larger deviations inflation targeting becomes the favourable policy.[47] Yetman (2006) extends the analysis

Riksbank left the Gold standard and adopted a price level target in the beginning of the 1930s in order to fight the deflationary spiral following the Great Depression. The formal objective of the policy was to revert any deviations in the price level from the target set equal to the price level in September 1931. The evidence presented in Berg and Jonung (1999) show that the policy succeeded in fighting deflation and reestablishing domestic purchasing power.

[46] The central bank's considerations are presented in Bank of Canada (2006) and in Amano et al. (2009). In Bank of Canada (2006) the central bank, for example, lists the improvement of monetary policy when the zero lower bound binds as an argument for adopting a price level target. In Amano et al. (2009) the central bank, for example, lists the uncertainty of whether the assumptions about the economy necessary for the policy to be optimal is satisfied as an argument against it.

[47] The analysis in Yetman (2003 and 2006) assumes a positive inflation target. Hence, a price level

to include the speed limit policy. The analysis finds that the speed limit policy also tends to outperform price level targeting when the private sector misperceives the target. Both Yetman (2003 and 2006) thus find that if the private sector fails to understand the price level target then price level targeting losses its superiority relative to inflation targeting.

The private sector misunderstanding the price level target may be a real problem for the central bank in short run. However, it is hard to imagine the private sector systematically making erroneous judgments about the target. Hence, one should expect that the private sector eventually learns about the target and starts forming expectations about the future based on the correct target. The analysis in Gaspar et al. (2007) sheds some light on this issue. If the private agents exhibit rational expectations then they would adjust their expectations immediately following a change to a new targeting regime. The analysis in Gaspar et al. (2007) instead investigates the transitional costs of moving from inflation targeting to price level targeting when the private agents fail to immediately adjust their expectations according to the new targeting regime. The analysis assumes that the private sectors expectations are formed by adaptive learning.[48] The overall conclusion of the analysis is that although the initial losses under price level targeting are greater than the losses under inflation targeting they will eventually converge towards the losses under the optimal commitment policy. Hence, although Yetman (2003 and 2006) finds that it may not be the preferred targeting regime if the private sector does not understand it, this remains an issue in the short run. However, if the private sector has to learn about the new regime the outcome will remain superior to the alternatives in the long run.

What if, however, the private sector attaches some positive weight to the probability that the central bank switches back to inflation targeting after having adopted price level targeting? One may motivate this by assuming that there exists a disagreement in the legislature responsible for appointing the central bank about what the desirable targeting regime is. The current legislature may favour a change to price level targeting, however, only by a narrow majority. A change in power in the legislature may then alter the view of what the preferred targeting regime is and decide to reappoint the inflation targeting central bank. This in turn impacts the formation of the private sector's expectations. Kryvtsov et al. (2008) look at the implications of switching from inflation targeting to price level targeting when the policy change suffers from imperfect credibility. The analysis assumes that the private sector initially attaches a positive probability to the event that the central bank switches back to inflation targeting following the adoption of

targeting central bank will aim at stabilising the price level around a target path for the price level. Yetman (2003 and 2006) thus motivates the analysis by assuming that the private sector has a different belief about the starting point of the path, hence, the initial price level target. The qualitative conclusions of the analysis does not, however, change by assuming a zero inflation target as done in this analysis.

[48]The adaptive learning assumption used in Gaspar et al. (2007) assumes that the private sector learns about the new targeting regime by observing the actual outcome for monetary policy and estimating an equation like $p_t = \theta_{11} p_{t-1} + \theta_{12} p_{t-2} + \epsilon_t$.

price level targeting, but that this probability converges to zero over time.[49] The intuitive consequence of adding this element to the model is similar to the case analysed in the previous section 4.4 when a reset of the price level target was a possibility. Again, it is the expectational channel which is affected. If the private sector values the risk of a policy reversal high then it will, to a greater extend, form its expectations about future inflation as if the central bank has an inflation target. The conclusions in Kryvtsov et al. (2008) are that this form of imperfect credibility reduces the favourability of price level targeting compared to inflation targeting. However, it is only when it takes ten or more quarters before the private sector fully trusts that the central bank will not switch back to inflation targeting that a change to price level targeting is not preferable.

Hence, both the analysis in Gaspar et al. (2007) and Kryvtsov et al. (2008) find that the long run gains from an improved monetary policy trade off through price level targeting tend to outweigh the transitional costs associated with a move from inflation targeting. Overall, the theoretical findings tend to support a change from inflation targeting to price level targeting even if the private sector does not fully comprehend the implications of the policy. However, the theoretical findings also find that a change may involve additional losses in the short-run.

4.6 Summary of the findings

The former analysis finds that when the central bank is forced to set monetary policy under discretion price level targeting can be used to replicate the optimal commitment solution to monetary policy derived in section 3.2. In the specific case, when there is no persistence in the cost-push shock process, price level targeting can perfectly replicate the optimal commitment solution. In all aspects, however, when the central bank is forced to act under discretion price level targeting improves monetary policy compared to inflation targeting by adding a stationary price level to the solution to monetary policy.

Furthermore, the analysis finds that price level targeting has additional leverage over inflation targeting when the economy is hit by a large demand shock which implies that the zero lower bound binds. This is because price level targeting enables the central

[49] Kryvtsov et al. (2008) consider two ways of modelling this assumption. If one assumes that δ_t now denotes the probability that the central bank stays with price level targeting then $(1-\delta_t)$ denotes the probability of a switch back to inflation targeting. Hence, $\delta_t = 1$ implies that a continuation of the price level targeting policy is fully credible. The analysis then considers one scenario where δ_t gradually adjusts towards one, following the law of motion given by $\delta_{t+1} = \delta_t + b\,(1-\delta_t)$, where $b \in [0,1]$ is the speed of convergence and another scenario where $\delta_t = \begin{cases} 0, & if\, t < T \\ 1, & if\, t \geq T \end{cases}$ and thus jumps discontinuously from zero to one at time T. Either way may be justified within the motivation used here. In the first scenario the minority opposition may gain confidence of the effectiveness of price level targeting over time, while in the second scenario the time T may constitute a shift in the legislature following an election which results in a full majority supporting price level targeting.

bank to optimally utilise the private sector's expectations and thus keep the interest rate operating procedure effective. The advantages of price level targeting may, however, be limited if the policy, for any reason, includes an escape clause as this will limit the gains from the expectational channel. Finally, the theoretical evidence point towards the conclusion that price level targeting remains the optimal policy in the long run even though it may involve short run transitional costs moving from, for example, inflation targeting to price level targeting.

5 Empirical investigation

The conventional way of analysing the optimality of monetary policy is by looking at the *ex ante* announced objectives for monetary policy. The *ex ante* objective is defined by the central bank's loss function and the level of commitment attached to minimising the loss function - see, for example, the central bank's objective under inflation targeting and price level targeting in equation (3.2) and (4.3) respectively. The formal analysis in section 3 and 4 found that the optimality of monetary policy in an economy where the private sector is forward-looking essentially depends on the statistical properties of the price level. Based on this result evaluating the *ex post* realisation of the monetary policy objective following a shock to the economy presents another way of making conclusions about the optimality of monetary policy. The *ex post* realisation of monetary policy is in this context defined by the statistical properties of the price level.

According to Schmidt-Hebbel (2009) a large number of central banks are characterised as inflation targeters. Using the former way of analysing monetary policy would then most likely lead to the general, and perhaps wrongful, conclusion that monetary policy does not resemble the optimal commitment. Furthermore, one may easily attempt to make the obvious conclusion based on the analysis in section 4 that the shift from inflation targeting to price level targeting considered by Bank of Canada described in footnote 46 is a rather straight forward decision. *However, if the Canadian price level is already stationary, then does this change necessarily improve on monetary policy?*

Hence, one should distinguish between the *ex ante* objectives for monetary policy announced by the central bank and the *ex post* realisation of the monetary policy objective when making conclusions about the optimality of monetary policy. The latter strategy motivates an empirical investigation of the characteristics of monetary policy based on the following empirical model for the price level

$$p_t = \theta_1 p_{t-1} + \epsilon_t \tag{5.1}$$

If θ_1 takes a value between zero and one then the price level follows a stationary path and monetary policy obtains the characteristics of the optimal commitment. If θ_1, on the other hand, takes the value one then monetary policy allows shocks to persist and the price level is non-stationary. Hence, if this relation is stationary then there is no need to reconsider the delegation of monetary policy. To further validate this empirical strategy recall the assumption made about the delegation of monetary policy in section 2. Following this assumption, the statistical properties of the price level are essentially a result of political decisions.

The following section will apply this empirical strategy to data on the price level in Australia, Canada, the Euro Area, Japan, New Zealand, Norway, Sweden, Switzerland, UK and the US. Conducting the empirical investigation on a rather broad sample of countries enables a more general conclusion about the optimality of monetary policy.

In order to be able to make any conclusions about the optimality of monetary policy based on this empirical strategy it is, however, necessary to assume that the model introduced in section 2 is a good characterisation of the economies in the sample.[50] Furthermore, it is necessary to assume that the structural parameters of the model and the distribution of the cost-push shocks are unchanged over the sample period. The validity of the latter assumption is weaker over larger sample periods, while the assumption may be more robust on a shorter sample.

5.1 Central bank objectives

Before investigating the statistical properties of the price level the objectives for monetary policy announced by the ten central banks in the sample are reviewed.

Following section 2.2, a central bank's objective is characterized by its loss function and the level of commitment attached to minimising the loss function. In reality, however, objectives are not formulated as precise and rigid as, for example, the inflation targeting objective in (3.1) is. The examination of the objectives of the ten central banks is not in any way final, but should be viewed as a brief qualitative assessment of what the central banks have announced and how the literature has viewed it. Regarding commitment on future actions it is convenient to view commitment from two perspectives. Borrowing the definition from Ferrero and Secchi (2009), a commitment is viewed as being either quantitative or qualitative. Furthermore, a commitment is viewed as either conditional on, for example, future economic conditions or unconditional which, for example, implies that the central bank commits to a certain policy for a fixed period. Besides the references made below the assessment builds on the references to the central bank's websites listed in appendix 8.7.

The Reserve Bank of Australia operates in a flexible average inflation targeting regime following the definition in section 3.3. This point is made in the analysis in Nessén and Vestin (2005) and confirmed by the official announcement of the central bank's objective which among other things state that the central bank should focus on *"...keeping consumer price inflation between 2 and 3 per cent, on average, over the cycle."*. The announcement does not, however, make any explicit definition of the length of a "cycle".

[50] One important distinction between the foundation underlying the empirical model in (5.1) and a number of the economies in the sample is that the economies in the sample may be characterised as open economies. Clarida et al. (2001) show that the optimal commitment solution carries over to the open economy where, however, the central bank only reacts to changes in domestic prices. There may be quantitative differences in the coefficients of the solution depending on the degree of openness of the economy.

Bank of Canada explicitly labels its monetary policy regime *inflation-control targeting.* Hence, it is safe to conclude that the Bank of Canada operates in an inflation targeting regime. Additionally, Bank of Canada has recently made a quantitative, conditional commitment about future monetary policy. That follows from the announcement in Bank of Canada (2009) *"Conditional on the outlook for inflation, the target overnight rate can be expected to remain at its current level until the end of the second quarter of 2010 in order to achieve the inflation target."*.

Article 127 in European Central Bank (2000) states that the primary objective of the European Central Bank is *"...to maintain price stability."*. It, furthermore, mentions that the central bank without prejudice to this objective should aim at achieving full employment and balanced economic growth. The Governing Board of the European Central Bank furthermore defines price stability as *"...inflation rates below, but close to, 2% over the medium term."*. Hence, one may argue that the European Central Bank has an inflation target. Note, that Schmidt-Hebbel (2009) does not count the European Central Bank as an inflation targeter. In addition "full employment" alludes to an output preference, while "balanced economic growth" alludes to an output growth objective. Thus, the objective contains an element of the speed limit policy defined in section 3.3. Ferrero and Secchi (2009) find that the European Central Bank on more occasions has made qualitative commitments about future monetary policy by the use of a the so-called *vigilance code language,* which is found to signal the direction of future monetary policy.

Bank of Japan *"...implements monetary policy with the aim of maintaining price stability."*. The central bank does not elaborate further on the definition of price stability. In Bank of Japan (2011) the central bank, however, makes a reference to its assessment of long-run inflation, which may suggest that price stability should be defined as some sort of objective for inflation. Bank of Japan has on more occasions made conditional, quantitative announcements on future monetary policy. One example is from 2001 where Bank of Japan promised to keep the policy rate at zero until *"...the consumer price index (excluding perishables, on a nationwide statistics) registers stably a zero percent or an increase year on year.*". A similar recent example is found in Bank of Japan (2010) p. 1.

The Reserve Bank of New Zealand is *"...required to conduct monetary policy with the goal of maintaining a stable general level of prices."* which is defined in terms of an average inflation target formulated as *"...keep future CPI inflation outcomes between 1 per cent and 3 per cent on average over the medium term."*. It does not, however, make any explicit definition of the meaning of "the medium term". The central bank, furthermore, makes a quantitative, conditional announcement on future monetary policy by publishing a future path for the interest rate.[51]

[51] Ferrero and Secchi (2009) and Blomquist (2010) survey central banks that publish a future path for the interest rate. Among the ten central banks included in the sample Reserve Bank of New Zealand,

Monetary policy in Norway is *"...oriented towards low and stable inflation."* operationalised as a *"...target of monetary policy shall be annual consumer price inflation of approximately 2.5 per cent over time."*. Hence, Norges Bank is labelled an inflation targeter. Similar to Reserve Bank of New Zealand, Norges Bank also publishes a future path for the interest rate.

The Riksbank has an inflation target. In Riksbank (2011) the monetary policy objective is defined with the aim of *"...maintain price stability."* Price stability further requires the Riksbank *"...to keep the annual increase in the CPI at 2 per cent."*. The Riksbank also publishes an interest rate path.

The Swiss National Bank *"...shall ensure price stability."*. This objective is defined in article 5 in Swiss National Bank (2009). Price stability is, furthermore, defined as *"...a rise in the national consumer price index (CPI) of less than 2% per annum."*. The Swiss National Bank thus operates in an inflation targeting regime. As noted in Schmidt-Hebbel (2009) the Swiss National Bank does not define its monetary policy framework as inflation targeting. However, Schmidt-Hebbel (2009) also lists Switzerland as an inflation targeting country.

"...the Board of Governors and the Federal Open Market Committee should seek "to promote effectively the goals of maximum employment, stable prices,..", Federal Reserve (2005) p. 15. Hence, the Federal Reserve's objective alludes to an output preference and a price stability preference without further defining the meaning of "stable prices". Walsh (2003) argues that the Federal Reserve's monetary policy announcements with their emphasis on economic growth motivate the speed limit policy as a proper characterisation of the monetary policy objective. Woodford (1999b) argues that the inertial interest rate setting by the Federal Reserve indicates that it has a preference for interest rate smoothing. Ferrero and Secchi (2009) find that the Federal Reserve on more occasions has made conditional, qualitative commitments about future monetary policy. Moreover, the Federal Reserve has recently made conditional, quantitative commitments about future monetary policy by the use of the so-called *extended period language*. A recent example is found in Federal Reserve (2011b) *"The Committee currently anticipates that economic conditions–including low rates of resource utilization and a subdued outlook for inflation over the medium run–are likely to warrant exceptionally low levels for the federal funds rate at least through mid-2013."*.

Bank of England's objective for monetary policy *"...is to deliver price stability – low inflation – and, subject to that, to support the Government's economic objectives including those for growth and employment."*, where the central bank further defines "price stability"

Norges Bank and Riksbank are publishing an interest rate path. According to the evidence in Blomquist (2010) these three central banks are among only five central banks to currently use this approach. The two others are Czech National Bank and Bank of Israel.

in terms of a target for inflation. Hence, besides targeting inflation this objective also draws from the speed limit policy.

5.2 Empirical test

The former analysis assumed that the social optimal rate of inflation was zero. However, the ten central banks included in the sample all aim for a positive rate of inflation [52] which implies an upward sloping trend in the price level. Furthermore, section 3.2 suggested that the central bank may adopt the optimal commitment from a timeless perspective and ignore the arbitrary t_0. If p_0 is normalised to zero then this would involve a non-zero constant term.

Hence, the empirical model for the price level should take account of this. Equation (5.1) is therefore augmented with a constant term and a trend to give

$$p_t = \vartheta + \gamma t + \theta_1 p_{t-1} + \epsilon_t \tag{5.2}$$

If the coefficient, θ_1, is between zero and one then the price level follows a trend stationary path with a starting point in $p_0 = \vartheta$ and with a slope of $\bar{\pi} = \gamma$. If $\theta_1$equals one then, on the other hand, the price level follows a random walk with a drift.

5.2.1 Test hypothesis

Since the interest in relation to the *ex post* realisation of monetary policy, is the stationary properties of (5.2) a proper way to empirically investigate the optimality of monetary policy is to apply the augmented Dickey-Fuller unit root test on(5.2). An outline of the test is found in Nielsen (2008).[53] The test considers the following null hypothesis

$$H_0 : \ \theta_1 = 1 \tag{5.3}$$

which implies that (5.2) follows a random walk with a drift. It is evaluated against the alternative, $H_A : \ -1 < \theta_1 < 1$, which implies that (5.2) follows a trend stationary path.

An equivalent version of the test considers the null, $H_0 : \ \nu = 0$, against the alternative, $H_A : \ -2 < \nu < 0$, on the difference equation, $\Delta p_t = \vartheta + \gamma t + \nu p_{t-1} + \epsilon_t$, where $\nu = \theta_1 - 1$.

[52] Table 5.2 includes the positive inflation targets for the eight inflation targeting central banks in the sample. The two remaining central banks, the Federal Reserve and Bank of Japan do not announce an explicit inflation target. Instead they published an assessment of long-run inflation. In Federal Reserve (2011a) p. 43 the Federal Reserve's long-run projection for inflation was 1.7-2.0%. In Bank of Japan (2011) p. 13 the board members of Bank of Japan regard an inflation around 1% consistent with price stability.

[53] The test is outlined for an autoregressive process of order one characterising the evolution of the price level. This is only for simplicity. When applying the test autoregressive processes of higher order are also considered in order to secure that ϵ_t is random and i.i.d..

If $\nu = \theta_1 - 1 = 0$ then γt is accumulated to produce a quadratic trend. To avoid this, the joint hypothesis, $H_0^\star : \nu = \gamma = 0$, is tested using the likelihood ratio test, $LR(\nu = \gamma = 0) = -2(\log L_0 - \log L_A)$. The test statistic follows a Dickey-Fuller distribution that allows for a constant term and a trend. Rejecting the null will provide evidence to support the conclusion that (5.2) exhibits the statistical property of trend stationarity and that monetary policy hereby obtains the characteristics of the optimal commitment.

In this case, the statistical properties of (5.2) can be described as follows

$$p_t = p_t^{STA} + \vartheta + \gamma t \tag{5.4}$$

where p_t^{STA} denotes the stationary part of the price level and ϑ together with γ denotes the starting point and the slope of the trend respectively.

Inference on the starting point in (5.4) can be used to test if monetary policy is optimal from a timeless perspective. Following the definition in section 3.2, a policy which is optimal from a timeless perspective does not depend on the arbitrary t_0. If monetary policy on the other hand depends on the arbitrary t_0 then the starting point of the trend should equal the initial price level. Testing this corresponds to testing the null hypothesis

$$H_0 : \vartheta = p_0 \tag{5.5}$$

against the two-sided alternative, $H_A : \vartheta \neq p_0$. When p_0 is normalised to zero then the null corresponds to the standard exclusion restriction on the constant term. Rejecting the null will indicate that monetary policy is optimal from a timeless perspective. However, a failure to reject the null does not necessarily imply the opposite. The central bank may very well find it optimal to choose p_0 as the starting point when implementing the policy from a timeless perspective.

Inference on the slope in (5.4) can be used to investigate if the slope of the trend in the price level corresponds to the central bank's target for inflation. It is of course only relevant to test this restriction for central banks that announce an explicit inflation target. However, assuming that the central bank's inflation target equals the social optimal rate of inflation in the economy then it should involve a better outcome for monetary policy if the slope of the trend in the price level equals the inflation target. If the opposite holds then one may argue that it involves a constant social loss in the long run as actual inflation deviates from the social optimal level.

The relevant null hypothesis depends on how the inflation target is formulated. If the target is announced as a point target[54] then it is apparent to test the null

$$H_0:\ \gamma = \bar{\pi} \tag{5.6}$$

The null is evaluated against the two-sided alternative, $H_A:\ \frac{\gamma}{1-\theta_1} \neq \bar{\pi}$. If the target instead specifies an upper bound for inflation[55] then the relevant null becomes

$$H_0:\ \gamma < \bar{\pi} \tag{5.7}$$

which is evaluated against the alternative, $H_A:\ \gamma \geq \bar{\pi}$. Finally, the central bank may announce a target range for inflation.[56] In this case it is relevant to test the null

$$H_0:\ \bar{\pi}^{lower\,bound} \leq \gamma \leq \bar{\pi}^{upper\,bound} \tag{5.8}$$

against the alternative, $H_A:\ \gamma < \bar{\pi}^{lower\,bound}\ or\ \gamma > \bar{\pi}^{upper\,bound}$.

Rejecting the respective null hypothesis will then provide evidence to support the conclusion that the central bank is setting monetary policy to stabilise the price level around a path that is either growing too fast or too slow compared to the social optimal level of inflation.

Table 5.1 summarises the test strategy and its implication for monetary policy.

	Reject (5.3)		Not reject (5.3)
	Reject (5.5)	Not reject (5.5)	
Reject (5.6), (5.7) or (5.8)	Optimal timeless commitment	Optimal commitment	
Not reject (5.6), (5.7) or (5.8)	Optimal timeless commitment, trend equals inflation target	Optimal commitment, trend equals inflation target	-

Table 5.1: Test summary

5.2.2 Data

To carry out the tests summarised in table 5.1 data on the monetary policy relevant price level index, a relevant sample start and an inflation target, if the central bank has

[54]Bank of Canada and Bank of England announce a point target and a symmetric tolerance band around the target. In this analysis it will be assumed that they find any deviation from the point target undesirable.

[55]This hypothesis implicitly assumes that the central bank is satisfied as long as inflation remains below the upper limit. It should, however, be noted that the European Central Bank, which announces and upper limit for inflation, actually aims at an inflation below, but close to the upper limit.

[56]This hypothesis implicitly assumes that the central bank finds all values of inflation within the range equally desirable.

announced one, for the countries in the sample is needed. Table 5.2[57] provides an overview of the data.

Country	Start date, t_0	Inflation target, $\bar{\pi}$	Price level index, p
Australia	1993:Q3	2-3%	CPI
Canada	1996:M1	2%	CPI
Euro Area	1999:M1	$<$ 2%	CPI
Japan	1998:M4	-	CPI ex. fresh food
New Zealand	1991:Q1	0-2% (1991:Q1) 0-3% (1997:Q1) 1-3% (2002:Q3)	CPI
Norway	2002:M1	$2\frac{1}{2}$%	CPI
Sweden	1995:M1	2%	CPI
Switzerland	2000:M1	$<$ 2%	CPI
UK	1992:M10	$2\frac{1}{2}$% (1992:M10) 2% (2004:M1)	RPIX (1992:M10) CPI (2004:M1)
US	1979:M8	-	Core-CPI

Table 5.2: Data

For all ten central banks the sample start is chosen at the time when the current monetary policy regime was adopted. As mentioned above, the statistical properties of the price level depends on the monetary policy objective. Applying the test to the period of the current regime enables the possibility of making conclusions about current monetary policy and assess whether current monetary policy is optimal from a timeless perspective. The samples chosen may, however, be subject to breaks or shifts which can influence the test results. This problem is discussed next. For the inflation targeting central banks the sample start is chosen at the time when the inflation targeting policy took effect. For Canada, however, the first three years of disinflationary targeting[58] has been disregarded. The start date for Japan is chosen at the time when the new Bank of Japan act came into effect.[59] And finally for the US, the start date is set at the time Paul Volcker took over as chairman of the Federal Reserve Board of Governors.[60]

The inflation targets listed in table 5.2 correspond to the official announced inflation

[57]The information in the table has been retrieved from the central bank's websites. See appendix 8.7 for a list of references. The price level time series have been collected from the national statistical offices using Macrobond Financial

[58]Disinflationary targeting refers to the special version of inflation targeting, where the central bank targets a downward sloping path for inflation which is gradually converging towards a fixed target corresponding to low inflation. Besides Bank of Canada, this policy has also been used by National Bank of Poland, Czech National Bank and the Turkish Central Bank among others as an effective means for bringing down inflation to a low an stable level.

[59]The Bank of Japan Act was finalised in June 1997 with effect of April 1998.

[60]This date is chosen as it is viewed as starting point of the focus on gaining price stability in US monetary policy. Orphanides (2006) analyses US monetary policy under the so-called *Volcker-Greenspan era*.

targets. Note, that Bank of Japan and the Federal Reserve do not announce an official inflation target. Hence, the test on the restriction on the trend slope is not carried out for these two central banks. Note further, that in the group of central banks with inflation targets Reserve Bank of Australia and Reserve Bank of New Zealand have a target specified as a range, the European Central Bank and the Swiss National Bank have an asymmetric target where the upper limit to inflation is announced and the remaining four, Bank of Canada, Norges Bank, the Riksbank and Bank of England all have announced a point target for inflation. Also note, that the Reserve Bank of New Zealand and Bank of England have changed the target during the period considered.

Finally, the price level indices used are the monetary policy relevant indices. For the inflation targeting central banks this corresponds to the index which is used to calculate the target relevant inflation. For Japan and the US the index used is the index the central bank refers to when it announces monetary policy. Note, that Bank of England changed the monetary policy relevant price index in the period considered. For UK a composite index is thus constructed as a merger and proper rebasement of the two indices in the period where the change took effect. Also note, that the statistical offices in Australia and New Zealand publish price level data on a quarterly basis.

5.2.3 Breaks and shifts

The statistical properties of the empirical model in (5.4) depends on the monetary policy objective. Hence, if the objective changes then that change may affect the statistical properties of the price level. Consequently, this may determine whether or not the null in (5.3) is rejected, thus, whether or not the price level is stationary. It is therefore important to take account of events which may impact the monetary policy objective and lead to breaks or shifts in (5.4). Restricting the sample period to the period where the current monetary policy regime has been in place limits the number of potential shifts and breaks in (5.4).

This does not mean, however, that (5.4) may not be subject to exogenous breaks or shifts. Table 5.3[61] contains a list of dates where possible exogenous breaks or shifts may have occurred. The table identifies three sources of exogenous breaks and shifts. The first is a change in the policy makers at the central bank. Column one in table 5.3 includes a list of dates when there has been a change in policy makers at one of the central banks in the sample. The table shows that this event has occurred in all the central banks one or more times during the respective sample period. The second source of exogenous breaks and shifts is a change in the variables included in the central bank's objective for monetary policy. Column two in table 5.3 provides the only example of such an event in the sample.

[61] The table is constructed using information collected from the central bank's websites. See appendix 8.7 for a list of references.

It was when Bank of England changed the monetary policy relevant price index in 2004. The third source is a change in the monetary policy target. As the third column in table 5.3 reports, this has happened two times in New Zealand and one time in UK. When Reserve Bank of New Zealand changed its target in 2002 the new target was formulated as an average inflation target. Finally, the fourth source of exogenous breaks and shifts is a change in the level of commitment to future monetary policy. Column four in table 5.3 shows that this has potential relevance for Reserve Bank of New Zealand, Norges Bank and the Riksbank. For all three central banks the dates reported mark the time when the respective central bank started publishing a path for the policy rate.[62]

Country	Change in policy makers	Change in variable	Change in target	Change in commitment
Australia	1996:Q3, 2006:Q3	-	-	-
Canada	2001:M2, 2008:M2	-	-	-
Euro Area	2003:M11	-	-	-
Japan	2003:M3, 2008:M4	-	-	-
New Zealand	2002:Q3	-	1997:Q1, 2002:Q3	1997:Q1
Norway	2011:M1	-	-	2005:M11
Sweden	2003:M1, 2006:M1	-	-	2007:M2
Switzerland	2001:M1, 2003:M7, 2007:M5, 2010:M1	-	-	-
UK	1993:M7, 2003:M7	2004:M1	2004:M1	-
US	1987:M8, 2006:M1	-	-	-

Table 5.3: Potential breaks or shifts

The four sources of potential exogenous breaks and shifts listed in table 5.3 may affect all three statistical properties of (5.4). The stationary properties of the price level may change following a change in the policy makers at the central bank. A change in policy makers may not alter the official announced monetary policy objective, although table 5.4 does provide one example of this, namely New Zealand in 2002. However, it may have an effect on the relative weight put on the variables in the objective which is assumed to be closely connected to the preferences of the policy makers at the central bank. In turn this may affect the stationary properties of the price level. Even without any changes to the loss function the stationary properties may be affected. This can happen if the central bank changes the way it commits to the future. Section 3.2 describes well the importance of commitment for monetary policy. Furthermore, the stationary properties of (5.4) may also change following a change in either the variables or the targets in the

[62]Section 5.1 describes more examples of central banks making commitments about future monetary policy. These examples have, however, been ignored here as it is difficult to determine whether they constitute a permanent shift in the monetary policy objective or not and when the shift may have taken place. This does not, however, mean that they are not a potential source of an exogenous break or shift in the monetary policy objective. Just that they are difficult to control for.

central bank's loss function. The starting point and the slope of the trend may also be affected by all four sources of exogenous breaks and shifts in table 5.3. The timeless perspective to monetary policy, for example, justifies changing either the starting point or the trend slope if needed.

One obvious way to take account of the impact of a potential break or shift is to augment the empirical model in (5.2) with a well specified dummy variable which corrects for the effects of a break or shift in (5.4). There are, however, two problems related to this approach. One is that the distribution under the null of a unit root in (5.2) including additional deterministic terms is highly complex. And the complexity increases with the number of dummy variables included.[63] Another problem is that this approach is not very robust to errors in the exogenous break dates listed in table 5.3. Even though the source of a potential break or shift occurred at either one of the dates listed in table 5.3 the actual break or shift may have happened at a different time. Furthermore, this approach fails to take account of changes in the stationary part of (5.4). A more robust way of taking account of potential breaks or shifts is by restricting the estimation of (5.2) to a shorter sample which eliminates the period prior to the potential break or shift. The drawback of this approach is that it reduces the number of observations in the estimation period. Fewer observations imply a less consistent estimation and thus a more uncertain outcome. Nonetheless, the latter approach is applied to check the robustness of the test results.

5.2.4 Test results

The augmented Dickey-Fuller test procedure follows the guidelines in Nielsen (2008). First, the number of lags in the empirical model given by (5.2) is determined through a standard general-to-specific estimation procedure to assure that the model is well specified. Second, the test statistic under the null (5.3) of a non-stationary price level is derived. Table 5.4 reports the results.

Country	Model	LR-test	Country	Model	LR-test
Australia	AR(1)	-2.05	Norway	AR(2)	-3.08
Canada	AR(2)	-3.41*	Sweden	AR(3)	-2.48
Euro Area	AR(2)	-2.35	Switzerland	AR(2)	-4.51**
Japan	AR(2)	-2.40	UK	AR(1)	-2.68
New Zealand	AR(1)	-2.83	US	AR(2)	-9.30**
**significant at 1% level, *significant at 5% level					

Table 5.4: Unit root test

[63] See Perron (1989) and Byrne and Perman (2007), among others, on the topic of testing for unit roots with structural breaks and shifts.

The null of a non-stationary price level is rejected on a 1% significance level for the US and Switzerland and on a 5% significance level for Canada. Hence, the empirical evidence point towards the conclusion that Bank of Canada, the Swiss National Bank and the Federal Reserve have conducted a monetary policy which resembles the optimal commitment. In the case of Norway, the null is almost rejected on a 10% significance level. The sample for Norway, however, is the shortest among the ten central banks. The test statistic is biased towards non-rejection for shorter samples though. Hence, it cannot be completely ruled out that the monetary policy set by Norges Bank has resembled the optimal commitment.

To test the robustness of the results in table 5.4 the test procedure is repeated on shorter samples starting after the exogenous break dates listed in table 5.3. When performing the test on reduced samples a trade off between eliminating the impact of possible breaks and shifts and producing consistent results arises. Table 5.5 therefore does not report test results on samples starting after 2006:M1.

Country	Sample start	Model	LR-test
Australia	1996:Q3	AR(1)	-3.63*
Canada	2001:M1	AR(2)	-3.56*
Euro Area	2003:M11	AR(2)	-1.85
Japan	2003:M3	AR(2)	-2.55
Norway	2005:M11	AR(2)	-3.11
New Zealand	1997:Q1	AR(1)	-3.65*
Sweden	2006:M1	AR(2)	-2.20
Switzerland	2003:M7	AR(2)	-3.79*
UK	2004:M1	AR(1)	-2.92
US	2006:M1	AR(1)	-4.24**
**significant at 1% level, *significant at 5% level			

Table 5.5: Unit root test on restricted sample

On the reduced samples the null of a non-stationary price level is rejected on a 1% significance level for the US and on a 5% significance level for Australia, Canada, New Zealand and Switzerland. The evidence thus point towards the conclusion that the monetary policy set by the Reserve Bank of Australia started resembling the optimal commitment policy following the change in the policy makers at the central bank in 1996. The evidence for New Zealand indicates that the Reserve Bank of New Zealand has set a policy which obtains the characteristics of the optimal commitment after the central bank changed inflation target and began publishing a path for the interest rate in 1997.

For Japan, it should be noted, that the reduced sample estimation suggests exclusion of the trend. For Japan, it turns out that the null of a non-stationary price level is almost rejected on a 10% significance level.[64] Hence, there is weak evidence to support the conclusion that the change in policy makers in 2003 changed the objectives of Bank of Japan in such a way that the outcome for monetary policy started resembling the optimal commitment.

In the case of Norges Bank the significance level at which the null is rejected is marginally improved when the test is applied on the reduced sample. This is a weak indication that the decision to start publishing a path for the interest rate in 2005 had an impact on the outcome for monetary policy. Again, it is important to keep in mind that the shorter sample biases the results towards a non-rejection of the null.

There were not found any significant evidence pointing towards a rejection of the null in the case of the Euro Area, Sweden and UK.

For the seven countries, including Japan and Norway, where the evidence supports a stationary price level, the coefficients of (5.4) is then estimated based on the estimation periods listed in table 5.5. The estimated coefficients and standard errors are reported in table 5.6 together with the level of significance of the null on the constant term in (5.5) and the relevant null on the trend slope in (5.6), (5.7) or (5.8). Note that since there were found no evidence of the significance of the trend slope in the Japanese price level it has been disregarded in the estimation.

Country	ϑ^{EST}	Significance level of (5.5)	γ^{EST}	Significance level of (5.6), (5.7) or (5.8)
Australia	-0.043093 (0.004298)	$<$ 0.1%	0.007210 (0.000091)	99.9%
Canada	0.011776 (0.002654)	$<$ 0.1%	0.001637 (0.000021)	15.7%
Japan	0.001620 (0.000714)	2.6%	-	-
Norway	-0.005375 (0.003645)	14.5%	0.001943 (4.43E-05)	0.2%
New Zealand	-0.038390 (0.005934)	$<$ 0.1%	0.006145 (0.000107)	99.9%
Switzerland	0.005552 (0.002455)	2.3%	0.000810 (0.000026)	99.9%
US	0.008579 (0.011070)	44.1%	0.001470 (0.000032)	-

Table 5.6: Estimated coefficients of (5.4)

The exclusion restriction on the constant term is rejected on a 1% significance level for Australia, Canada and New Zealand and on a 5% significance level for Japan and Switzer-

[64]When the Dickey-Fuller test is only augmented with a constant the distribution under the null moves to the right which implies rejecting the null for lower test statistics.

land. In the case of Norway and the US the null cannot be rejected. This indicates that Reserve Bank of Australia, Bank of Canada, Bank of Japan, Reserve Bank of New Zealand and the Swiss National Bank has implemented the optimal policy from a timeless perspective. It also indicates that the implementation of the optimal monetary policy by Norges Bank and Federal Reserve depends on the arbitrary t_0, but this does not, as mentioned above, rule out that monetary policy is implemented from a timeless perspective. Furthermore, it is important to stress that the results depend entirely on the assumption that the implementation of the optimal policy occurred at the sample start.

For Canada, the estimated annual increase in the price level trend is 2.0%. This is not significantly different from the official inflation target announced by Bank of Canada of 2%. The estimated slope of the Australian price level trend corresponds to an annual inflation rate of 2.9%. It cannot be rejected that it is located in the range of 2-3% which is the official target range for inflation set by Reserve Bank of Australia. The estimated slope of the price level trend in New Zealand corresponds to an annual inflation rate of 2.5%. It can also not be rejected that it is located in the official target range for inflation of 1-3% set by the Reserve Bank of New Zealand. The estimated slope of the price level trend in Switzerland corresponds to an annual rate of inflation of 1.0% and it is not found to be significantly different from the Swiss National Bank's inflation target. Hence, the empirical evidence indicates that the optimal monetary policy set by Bank of Canada, the Reserve Bank of Australia, the Reserve Bank of New Zealand and the Swiss National Bank respectively is in accordance with the inflation target. The estimated slope of the Norwegian price level trend corresponds to an annual rate of inflation of 2.3%, which is significantly different on a 1% level from the inflation target of $2\frac{1}{2}$% set by Norges Bank. Hence, the evidence point towards the conclusion that the optimal monetary policy conducted by Norges Bank is not in accordance with the announced inflation target.

Figures 5.1-5.10 depict the actual price levels for all ten central banks. For the seven central banks which have conducted a policy that exhibits the characteristics of the optimal commitment the estimated trend is included in the illustration.

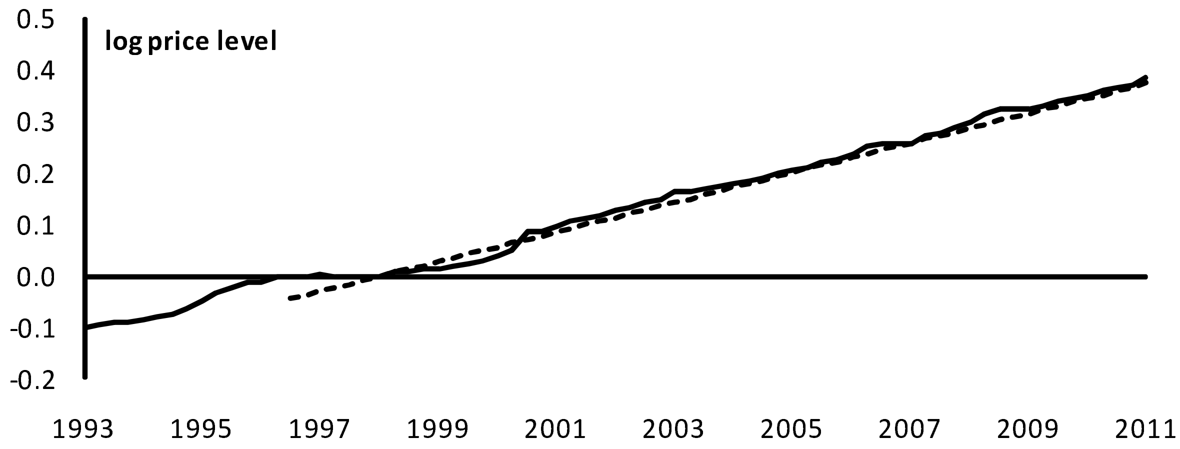

Figure 5.1: Australia

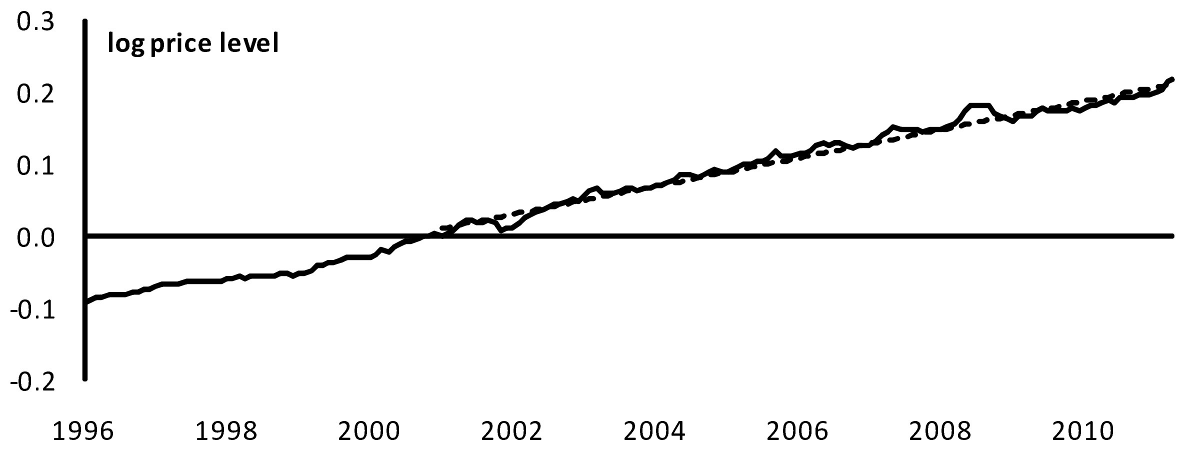

Figure 5.2: Canada

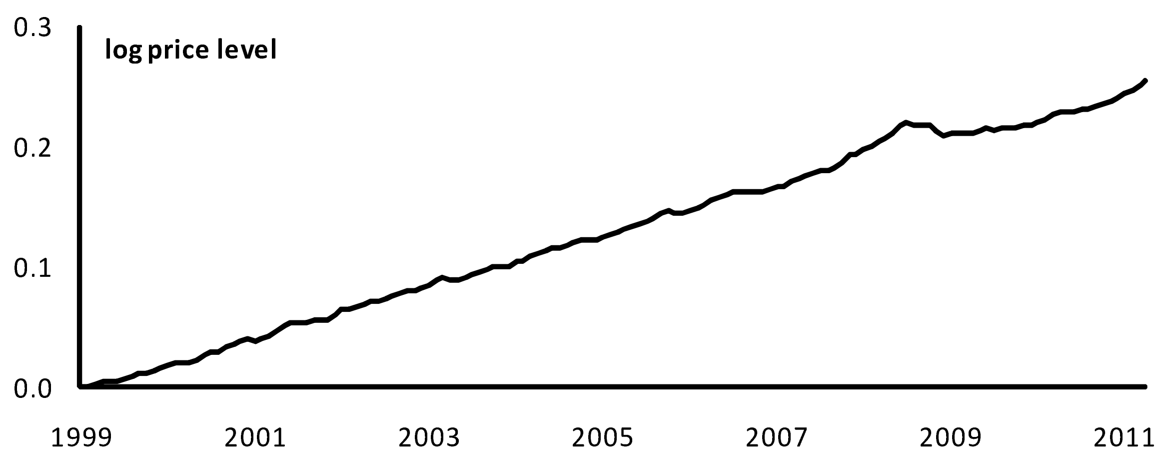

Figure 5.3: Euro Area

Figure 5.4: Japan

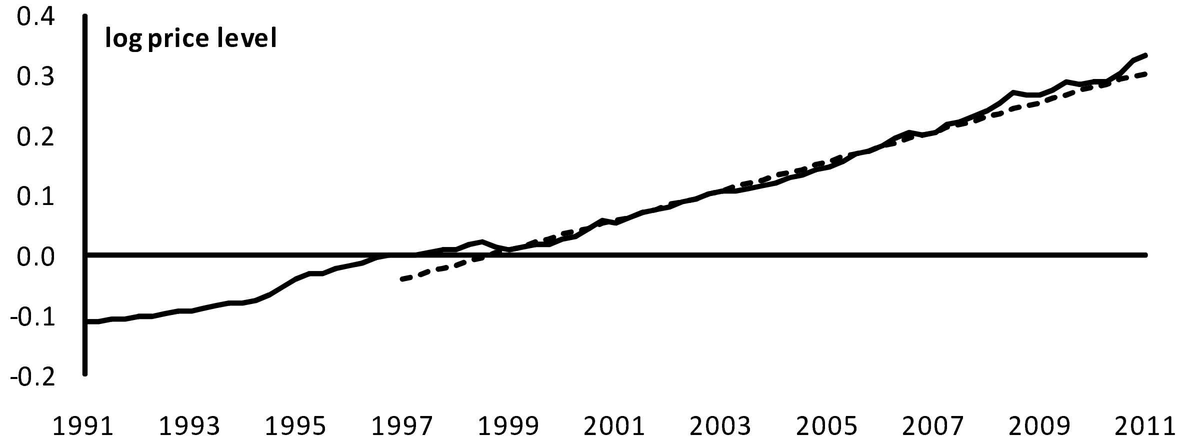

Figure 5.5: New Zealand

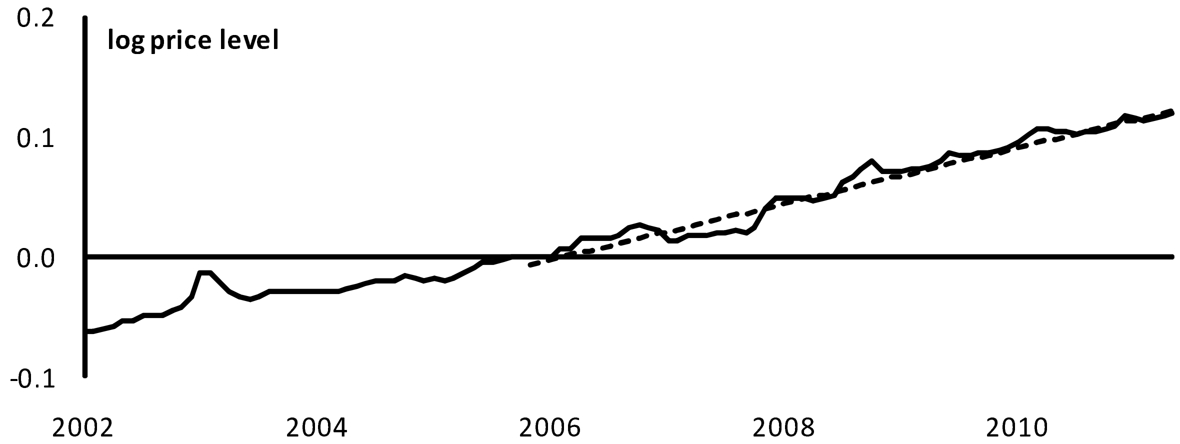

Figure 5.6: Norway

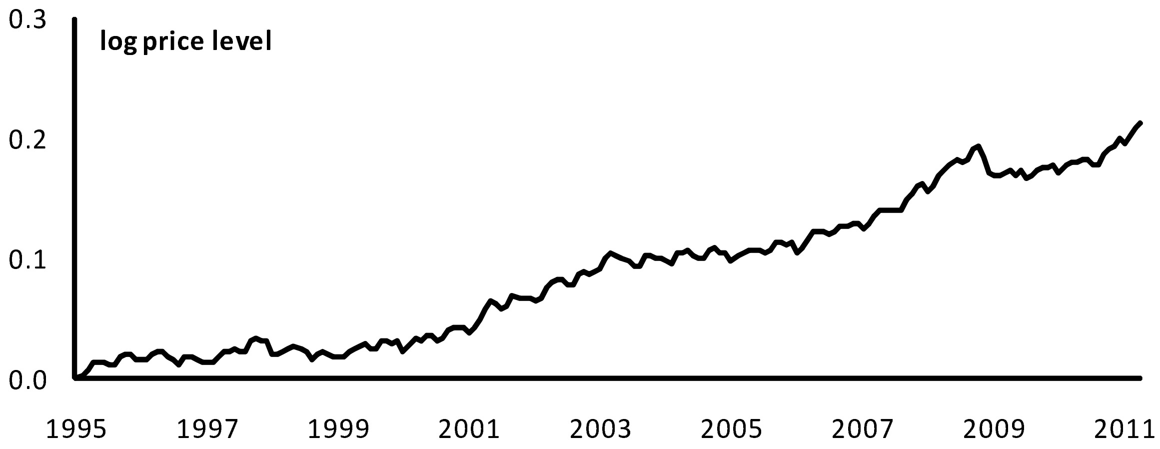

Figure 5.7: Sweden

Figure 5.8: Switzerland

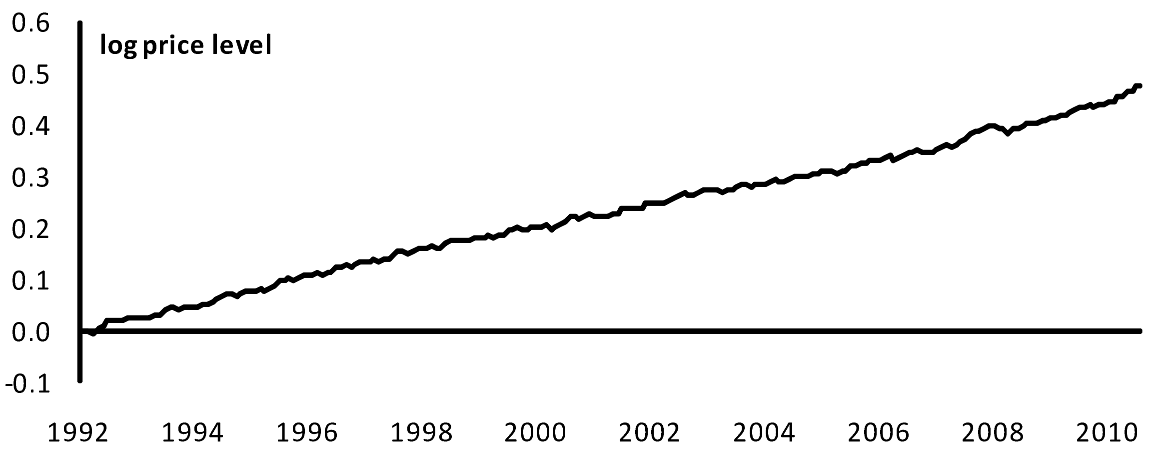

Figure 5.9: UK

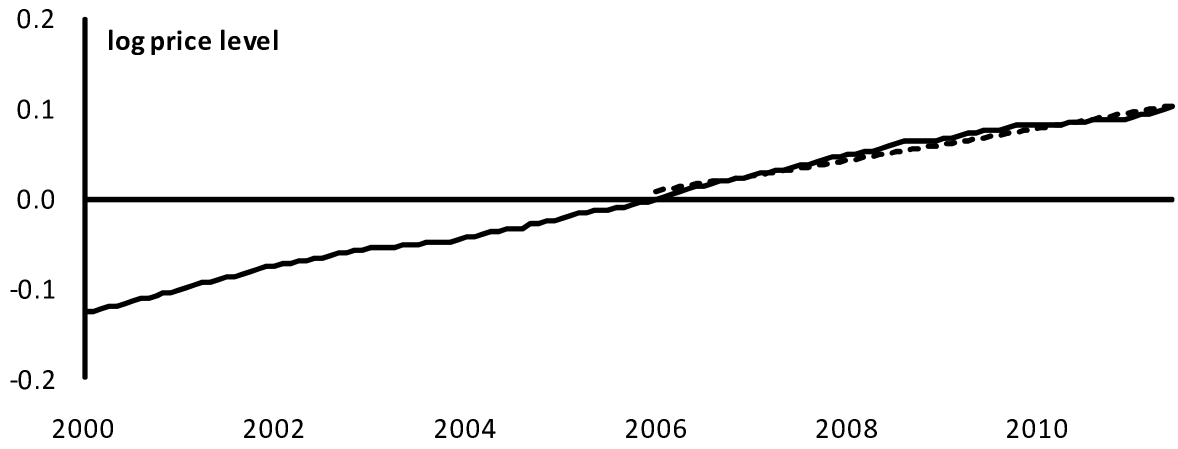

Figure 5.10: US

Table 5.7 summarises the empirical findings. The evidence supports the conclusion that monetary policy in Australia, Canada, Japan, New Zealand, and Switzerland has resembled the optimal timeless commitment. In Australia, Canada and New Zealand the policy has furthermore been in accordance with the inflation target. In Norway and the US monetary policy is found to have resembled the optimal commitment, but not from a timeless perspective, however, noting the uncertainties attached to the latter result. It is, furthermore, worth noting that this conclusion only holds for past realisations of monetary policy and does not necessarily provide any indication about the future development in the price level and thus the future optimality of monetary policy. Monetary policy in the Euro Area, Sweden and in UK has not been found to have resembled the optimal commitment policy.

<table>
<tr><th rowspan="2"></th><th colspan="2">Reject (5.3)</th><th rowspan="2">Not reject (5.3)</th></tr>
<tr><th>Reject (5.5)</th><th>Not reject (5.5)</th></tr>
<tr><td>Reject (5.6), (5.7) or (5.8)</td><td>Japan</td><td>Norway, US</td><td rowspan="2">Euro Area, Sweden, UK</td></tr>
<tr><td>Not reject (5.6), (5.7) or (5.8)</td><td>Australia, Canada, New Zealand, Switzerland</td><td>-</td></tr>
</table>

Table 5.7: Summary of test results

5.2.5 Uncertainties

The previous investigation failed to find any evidence that monetary policy in UK has resembled the optimal commitment. A similar analysis in Ruge-Murcia (2009), however, reaches the opposite conclusion. In Ruge-Murcia (2009) Bank of England is found to have set a monetary policy which has resembled the optimal commitment. The obvious explanation for this difference is that the analysis in Ruge-Murcia (2009) is based on a shorter sample. If the unit root test for UK is performed on a sample which ends in 2007:M12 then the conclusion changes. The evidence point towards the conclusion that the monetary policy set by Bank of England during this period monetary has resembled the optimal commitment which furthermore indicates a break or shift in monetary policy in UK around this time. It is not, however, possible to identify the potential source of this break or shift from table 5.3. One explanation could be that Bank of England changed its monetary policy objective following the Financial Crisis which hit UK around this time. It is furthermore not possible to identify what this change has meant for monetary policy in UK since the sample starting in 2007:M12 is not long enough to get consistent estimates of the empirical model in (5.2). Hence, it is not possible to identify whether Bank of England has set a stationary price level in the period following 2007:M12 meaning that the failure to find evidence of a stationary price level over the entire sample period may be regarded a target reset or slope change.

The previous investigation also failed to provide any evidence that the Riksbank has set a stationary price level. The analysis in Ruge Murcia (2009) did not succeed in finding any evidence either. The analysis, however, failed to take account of the change in commitment in 2007:M2 made by the Riksbank. At this point the Riksbank started publishing a path for the interest rate. The sample starting at this date is too short to get consistent estimates and thus investigate if it has had any significant impact on the statistical properties of the Swedish price level. However, there may be some support found in the evidence from New Zealand and Norway. The results of the empirical test show that when the Reserve Bank of New Zealand and Norges Bank started publishing a path for the interest rate it changed the conclusion about the statistical properties of the price level in New Zealand and Norway respectively. Hence, the decision by the Riksbank to start publishing a path for the interest rate may also have implied a stationary price level.

5.3 Discussion of the findings

The empirical investigation in section 5.2 concluded that the statistical properties of the price level in Australia, Canada, Japan, Norway, New Zealand, Switzerland and the US obtains the characteristics of the optimal commitment. Comparing this conclusion with the review of the central bank objectives presents a potential anomaly. Of the seven central banks with a stationary price level, three of them are defined as inflation targeters, two as average inflation targeters and one may follow a speed limit policy. Hence, based on this comparison, one may conclude that the actual outcome for monetary policy does not match the outcome one would have predicted by inspection of the objectives. If the evaluation of monetary policy was only based on the review then one might arrive at the incomplete conclusion that monetary policy is in fact not optimal. Instead an anomaly arises, where one may argue that monetary policy is characterised by "non-optimal intentions, but optimal actions".

The three remaining central banks in the Euro Area, Sweden and UK, where the evidence did not point towards the conclusion that they had set a monetary policy which resembles the optimal commitment, are all three characterised as inflation targeting central banks in the review. Based on this comparison, one may arrive at the conclusion that the actual outcome corresponds to the outcome predicted by the review of the objectives. Hence, for these three central banks there may be a case for improving the outcome for monetary policy by replacing the current targeting regime with the price level targeting regime analysed in this thesis, while, however, noting the uncertainties attached to the results - as discussed in the previous section.

The implications for monetary policy in the first mentioned seven countries, however, depends on how current monetary policy is perceived. If expectations are rational and the central bank's announcement of its monetary policy objective is perceived to be fully credible then one would expect expectations to be formed based on the official objective for monetary policy. In the case of the three inflation targeting central banks in Canada, Norway and Switzerland, private expectations about future inflation are then based on the solution to inflation targeting in equation (3.5). However, monetary policy resembles the solution to the optimal commitment in equation (3.15). In this case, however, the central bank may not gain full advantage of the expectational channel connected to setting a stationary price level. To get a clear view of the implications for monetary policy one would have to use a model which allows for heterogeneous expectations or asymmetric information. Nevertheless, since the central bank is setting a monetary policy which resembles the optimal commitment solution, it may constitute a "free lunch" of merely announcing that it is doing so - for example by replacing the current targeting regime with the price level targeting regime analysed in this thesis.

One drawback of this approach is that expectations may in fact not be rational. Expectations may instead be formed by observing the actual outcome for monetary policy, for example, through adaptive learning - see Gaspar et al. (2007) and footnote 48 on adaptive learning when the solution to monetary policy is a stationary price level. In this case the anomaly presented above may have no implications for current monetary policy. In this case the central bank will gain full advantage of the expectational channel by setting a stationary price level. Thus, any changes to the monetary policy framework may not be needed. Replacing the current targeting regime with the price level targeting regime analysed in this thesis may, however, still constitute a "free lunch" and may further reduce the costs of learning and speed up the convergence to the rational expectations equilibrium. Further knowledge on this is, however, an issue for future research.

A recent empirical contribution to this discussion is found in Walsh (2009b). Walsh (2009b) constructs a counter factual scenario for US monetary policy during the recent Financial Crisis. In the counter factual scenario inflation expectations are instead formed as if the Federal Reserve had started setting a stationary price level prior to the Financial Crisis.[65] There is of course a great amount of uncertainty connected to a counter factual analysis. Nevertheless, the conclusion of the analysis in Walsh (2009b) is that the Federal Reserve would have been able to utilise expectations better if it had adopted a price level target prior to the Financial Crisis. Another recent empirical contribution is found in Cateau et al. (2009). Cateau et al. (2009) basically perform the analysis of Kryvtsov et al. (2008), see section 4.5 for a review of this analysis, on ToTEM, Bank of Canada's model for the Canadian economy. Hence, Cateau et al. (2009) investigate the specific gains for Canadian economy of adopting price level targeting when credibility is imperfect. The analysis concludes that there may be substantial gains if Bank of Canada adopts a price level targeting policy like the one analysed in this thesis. In the specific cases of Canada and the US there may thus be gains of replacing the existing regime with a price level targeting regime such as the one analysed in this thesis and since both central banks are already setting a stationary price level the transition may not be too difficult. If the US, a large open economy, and Canada, a small open economy, gain by adopting price level targeting, then one may argue that the recommendation carries over to the remaining eight countries in the sample (and other countries outside the sample as well).

[65] Walsh (2009b) constructs the counter factual scenario by assuming a target path for the price level to return increasing at an annual rate of 2% and assuming that it takes four quarters to return to the path. Hypothetical paths for inflation expectations are then constructed using different starting points and compared to the actual inflation expectations implied by the indexed 5-year Treasury bond.

6 Conclusion

The present thesis has revisited the arguments for and against a policy focusing on a price level target. There is currently no central banks operating a regime of price level targeting and only one central bank, Bank of Canada, is actively considering adopting the policy. In fact, historically, the Riksbank in the 1930s remains the only central bank to have had a price level target. Nevertheless, price level targeting presents the optimal way of utilising the forward-looking inflation expectations. The ability to utilise inflation expectations has become highly relevant following the experience of the Financial Crisis. A number of central banks are currently conducting monetary policy at very low nominal interest rates and have begun using unconventional tools of monetary policy. In this environment price level targeting presents a way of keeping the conventional interest rate operating procedure effective.

When economic agents are forward-looking the optimal commitment solution to monetary policy is history dependent and includes a stationary price level. Since the central bank is generally not assumed to be able to commit, this thesis has looked at alternative ways of implementing the optimal commitment solution, when the central bank is forced to act under discretion. Even though society is concerned with deviations in inflation and the output gap, it is optimal for the central bank to have a different objective for monetary policy. If the central bank instead is concerned with changes in the output gap, which is the core of nominal income growth targeting and the speed limit policy, then the outcome for monetary policy would be history dependent. However, if the central bank is concerned with price level stability, the outcome is both history dependent and includes a stationary price level and thus closely resembles the optimal commitment solution. In specific circumstances it is actually possible to perfectly replicate the optimal commitment solution through price level targeting.

Compared to inflation targeting, price level targeting presents a way of keeping the interest rate operating procedure effective, when the zero lower bound on the nominal interest rate binds. Inflation targeting anchors inflation expectations at the inflation target. This puts a lower constraint on the real interest rate. Under price level targeting inflation expectations depends negatively on the deviation of the actual price level from the target. Inflation expectations will therefore push the real interest rate downwards as long as the actual price level undershoots the target. Price level targeting is thus letting the market do part of the stabilisation. Price level targeting requires the central bank to revert all shocks to the price level. However, if a shock is caused by external factors, which the central has no control over, or if the shock is above a certain magnitude, then this reaction may be unwanted. One way for a price level targeting central bank to avoid reacting to specific shocks is to include an escape clause, which allows the central bank to ignore the shocks and let bygones be bygones. However, as this thesis has shown, an escape clause, which allows a price level targeting central bank to adopt the reaction of an inflation targeting central bank under given circumstances, will limit the effectiveness of the expectational channel. The central bank should therefore "target what it can hit" and maintain a stationary price level at all times. As mentioned, only one central bank has ever operated in a price level targeting regime. It is therefore fair to assume that it is associated with some transitional costs moving from, for example, inflation targeting to price level targeting as the economic agents may not fully comprehend the policy at first. However, even if this assumption holds, this thesis finds that it is still advantageous in the long run to adopt price level targeting.

Empirically, there exists a monetary policy anomaly. This thesis has evaluated monetary policy in a sample of ten major central banks. Reviewing what objectives the central banks announce for monetary policy reveals that the central banks in the sample can be characterised as some form of inflation targeters, average inflation targeters or following a speed limit policy. However, inspecting the statistical properties of the price level shows that the central banks in Australia, Canada, Japan, Norway, New Zealand, Switzerland and US have set a monetary policy which resembles the optimal commitment. This thesis therefore argues that monetary policy in these seven countries has been characterised by "non-optimal intentions, but optimal actions". The cost and gains of announcing a price level target for any of these seven central banks may therefore be limited depending on how expectations are formed. The remaining three central banks in the Euro Area, Sweden and UK are not found to have set a monetary policy which resembles the optimal commitment. Hence, these central banks may gain by adopting price level targeting.

This thesis has not considered practical issues concerning price level targeting. These include, among other things, what aggregate price level index the central bank should target. Recall, that escape clauses limit the advantage of price level targeting and it is thus even more important that the central bank target prices it can control. Another issue is whether the central bank should target a fixed price level or a drifting price level path. Some of the arguments for a positive inflation may not be available under price level targeting. As mentioned the expectational channel, for example, keeps monetary policy effective at the zero lower bound which removes the need of aiming for positive inflation. Finally, it may be worth considering issues such as, what horizon the central bank should aim at maintaining price level stability, how the central bank should communicate the price level objective and whether price level targeting should be implemented as price level forecast targeting - the equivalent to inflation forecast targeting. Furthermore, this thesis has not looked at the long-run advantage of greater price level certainty. This may involve an additional welfare improvement as it increases the certainty of the real outcome of nominal contracts. In all aspects, these issues are left for future research.

7 References

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8 Appendix

8.1 Micro foundation of the New Keynesian model

The following outline of the micro foundation of the model introduced in section 2 builds on Blomquist (2010) and Walsh (2010).

The households The economy consists of a large number of identical, optimising, infinitely living households. In this model it is assumed that households gain utility from consuming a basket of goods, holding money and leisure. In each period, t, the households therefore choose an optimal future path of the composite consumption good, C_t, real money holdings, $\frac{M_t}{P_t}$, and the supply of labour, N_t. This amounts to maximising the present discounted value of utility given by

$$E_t \sum_{i=0}^{\infty} \beta^i \left[u\left(C_{t+i}, \frac{M_{t+i}}{P_{t+i}}; \varpi^C_{t+i}, \varpi^M_{t+i} \right) - v\left(N_{t+i}; \varpi^N_{t+i} \right) \right]$$

The households are assumed to gain decreasing marginal utility of consumption and holding real money. u is therefore an increasing concave function in C_t and $\frac{M_t}{P_t}$. Furthermore, households are assumed to gain increasing marginal disutility of supplying labour. v is thus an increasing convex function in N_t. ϖ^C_{t+i}, ϖ^M_{t+i} and ϖ^N_{t+i} respectively denote exogenous preference shocks which reflects real shocks to the economy. The discount factor, $0 < \beta < 1$, denotes the household time preference.

The composite consumption good is defined as

$$C_t \equiv \left[\int_0^1 c_{j,t}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}}$$

where $\theta > 1$ is the price elasticity of demand for the individual goods. The households supply labour, n_j, to firm j, which produces good c_j. The household supply of labour to the different firms is given by $N_t = \int_0^1 n_{j,t} dj$.

The household decision problem is dealt with in two stages. First, the households minimise the costs of buying a given level of the composite good, C_t. Second, given the costs of buying a given level of C_t, the households choose the optimal value of C_t , M_t and N_t.

The first stage cost minimisation problem can then be stated as

$$\min_{c_{j,t}} \int_0^1 p_{j,t} c_{j,t} dj$$

subject to

$$\left[\int_0^1 c_{j,t}^{\frac{\theta-1}{\theta}} dj\right]^{\frac{\theta}{\theta-1}} \geq C_t \tag{8.1}$$

where $p_{j,t}$ is the price of $c_{j,t}$. The solution to this problem is found by solving the Lagrangian

$$\mathcal{L} = \int_0^1 p_{j,t} c_{j,t} dj - \psi_t \left(\left[\int_0^1 c_{j,t}^{\frac{\theta-1}{\theta}} dj\right]^{\frac{\theta}{\theta-1}} - C_t \right)$$

where ψ_t is the Lagrangian multiplier. The corresponding first-order condition can be written as

$$c_{j,t} = \left(\frac{p_{j,t}}{\psi_t}\right)^{-\theta} C_t \tag{8.2}$$

and the solution for ψ_t is

$$\psi_t = \left[\int_0^1 p_{j,t}^{1-\theta} dj\right]^{\frac{1}{1-\theta}} \equiv P_t \tag{8.3}$$

which is the aggregate price index for consumption. The demand for good j at time t can then be found by substituting equation (8.3) into (8.2) which gives

$$c_{jt} = \left(\frac{p_{j,t}}{P_t}\right)^{-\theta} C_t \tag{8.4}$$

One important thing to note from (8.4) is that the value of θ determines the degree of competitiveness in the economy. As $\theta \to \infty$ the different goods become closer substitutes and thus the individual firms will have less market power.

Using (8.4), the cost function in (8.1) can be written as

$$\int_0^1 p_{j,t} c_{j,t} dj = P_t C_t \tag{8.5}$$

Further using (8.5), the household budget constraint can then be expressed in real terms as

$$C_t + \frac{M_t}{P_t} + \frac{B_t}{P_t} = \left(\frac{W_t}{P_t}\right) N_t + \frac{M_{t-1}}{P_t} + (1 + i_{t-1}) \left(\frac{B_{t-1}}{P_t}\right) + \Pi_t \tag{8.6}$$

where B_t is the household holdings of one-period bonds, which pay the nominal rate of interest, i_t, W_t is labour income and Π_t is the real profits received from firms. The left hand side of (8.6) denotes the households real expenditure on consumption and desired money and bond holdings in period t. The right hand side of (8.6) is the households real wealth in period t determined by labour income, real money holdings, real interest on bonds and real profits.

The second stage of the household decision problem is then given by

$$\max_{C_t, \frac{M_t}{P_t}, N_t} E_t \sum_{i=0}^{\infty} \beta^i \left[u\left(C_{t+i}, \frac{M_{t+i}}{P_{t+i}}; \varpi^C_{t+i}, \varpi^M_{t+i} \right) - v\left(N_{t+i}; \varpi^N_{t+i} \right) \right] \tag{8.7}$$

subject to (8.6).

Assuming that households are exhibiting constant relative risk aversion the utility functions $u(\cdot)$ and $v(\cdot)$ are given by

$$u\left(C_t, \frac{M_t}{P_t}; \varpi^C_t, \varpi^M_t \right) = \frac{\left(\varpi^C_t C \right)^{1-\sigma}}{1-\sigma} + \frac{\upsilon}{1-\nu} \left(\frac{\varpi^M_t M_t}{P_t} \right)^{1-\nu} \tag{8.8}$$

$$v\left(N_t; \varpi^N_t \right) = \chi \frac{\left(\varpi^N_t N_t \right)^{1+\eta}}{1+\eta} \tag{8.9}$$

Using (8.8) and (8.9) in (8.7), the following Euler conditions are found which must hold in equilibrium along with the budget constraint in (8.6)

$$\left(\varpi^C_t \right)^{1-\sigma} C_t^{-\sigma} = \beta \left(1 + i_t \right) E_t \left[\left(\frac{P_t}{P_{t+1}} \right) \left(\varpi^C_{t+1} \right)^{1-\sigma} C_{t+1}^{-\sigma} \right] \tag{8.10}$$

which is the well-known Keynes-Ramsey rule for optimal intertemporal consumption allocation and

$$\frac{\upsilon \left(\varpi^M_t \right)^{1-\nu} \left(\frac{M_t}{P_t} \right)^{-\nu}}{\left(\varpi^C_t \right)^{1-\sigma} C_t^{-\sigma}} = \frac{i_t}{1+i_t} \tag{8.11}$$

which is the intratemporal optimality condition that sets the marginal rate of substitution between holding real money and consumption equal to the opportunity cost of holding money and finally,

$$\frac{\chi \left(\varpi^N_t \right)^{1-\eta} N_t^{\eta}}{\left(\varpi^C_t \right)^{1-\sigma} C_t^{-\sigma}} = \frac{W_t}{P_t} \tag{8.12}$$

which is the intratemportal optimality condition that sets the marginal rate of substitution between leisure and consumption equal to the real wage.

The firms The economy further consists of profit maximising firms which produce different goods. Each firm chooses the production plan which maximises profits given by

$$\Pi_{j,t} = \frac{P_{j,t}}{P_t} c_{j,t} - \frac{W_t}{P_t} N_{j,t} \tag{8.13}$$

conditional on the production function

$$c_{j,t} = Z_t N_{j,t} \tag{8.14}$$

where firm j uses the specific labour input, $N_{j,t}$, and the available technology, Z_t, to produce the specific good $c_{j,t}$, and conditional on the demand curve it faces, which is given by equation (8.4) and finally, the firms are not able to adjust prices each period. The price setting follows the Calvo staggered price setting discussed in section 2.1.

Each firm aims at minimising the production costs, $W_t N_{j,t}$. The cost minimisation problem facing the individual firm, expressed in real terms, is given by

$$\min_{N_{j,t}} \left(\frac{W_t}{P_t}\right) N_{j,t} \tag{8.15}$$

subject to the production function in (8.14). The solution to (8.15) is found by solving the Lagrangian

$$\min_{N_{j,t}} \left(\frac{W_t}{P_t}\right) N_{j,t} - \varphi_t \left(c_{j,t} - Z_t N_{j,t}\right) \tag{8.16}$$

where φ_t is the Lagrangian multiplier. The solution to (8.16) implies the following first-order condition

$$\varphi_t = \frac{W_t}{P_t Z_t} \equiv mc_t \tag{8.17}$$

which can be interpreted as the firm's real marginal costs, mc_t.

Using the production function in (8.14) and the real marginal costs in (8.17), the profit function in (8.13) can be rewritten to give

$$\Pi_{j,t} = \frac{P_{j,t}}{P_t} c_{j,t} - mc_t c_{j,t}$$

The firm then sets its individual price to maximise profits. Due to the staggered price setting the price setting problem is given by

$$E_t \sum_{i=0}^{\infty} \omega^i \Delta_{i,t+i} \left[\left(\frac{p_{j,t}}{P_{t+i}}\right) c_{j,t+i} - mc_{t+i} c_{j,t+i}\right] \tag{8.18}$$

where the discount factor is $\Delta_{i,t+i} = \beta^i \left(\frac{C_{t+i}}{C_t}\right)^{-\sigma}$ and ω^i denotes the probability that the firm will adjust its price. Equation (8.18) can be rewritten using the demand curve in (8.4) to give

$$E_t \sum_{i=0}^{\infty} \omega^i \Delta_{i,t+i} \left[\left(\frac{p_{j,t}}{P_{t+i}}\right)^{1-\theta} - mc_{t+i} \left(\frac{p_{j,t}}{P_{t+i}}\right)^{-\theta}\right] C_{t+i} \tag{8.19}$$

Since, all firms use the same production technology and face the same demand curve they are essentially identical and the only thing separating them is the time when they last adjusted their individual price. As in section 2.1 $p_t^{\star}$ denotes the optimal price chosen by firms able to adjust their price in period t. The first-order condition for equation (8.19) is then

$$E_t \sum_{i=0}^{\infty} \omega^i \Delta_{i,t+i} \left[(1-\theta)\left(\frac{p_t^\star}{P_{t+i}}\right) - \theta mc_{t+i}\right]\left(\frac{p_t^\star}{P_{t+i}}\right)^{-\theta} C_{t+i} = 0 \tag{8.20}$$

which can be rewritten using the definition of $\Delta_{i,t+i}$ stated above to give

$$\left(\frac{p_t^\star}{P_t}\right) = \left(\frac{\theta}{\theta-1}\right) \frac{E_t \sum_{i=0}^{\infty} \omega^i \beta^i C_{t+i}^{1-\sigma} mc_{t+i} \left(\frac{p_t^\star}{P_{t+i}}\right)^{\theta}}{E_t \sum_{i=0}^{\infty} \omega^i \beta^i C_{t+i}^{1-\sigma} \left(\frac{p_t^\star}{P_{t+i}}\right)^{\theta-1}} \tag{8.21}$$

The fraction, ω of firms not able to adjust their price in period t remains with their last period price which is equivalent to P_{t-1}. The aggregate price index in period t is thus given by

$$P_t^{1-\theta} = (1-\omega)\left(p_t^\star\right)^{1-\theta} + \omega P_{t-1}^{1-\theta} \tag{8.22}$$

Equation (8.21) and (8.22) can be log-linearised around a zero average inflation, steady-state equilibrium to give an expression for aggregate inflation[66]

$$\pi_t = \beta E_t \pi_{t+1} + \tilde{\kappa} \hat{mc}_t \tag{8.23}$$

where

$$\tilde{\kappa} = \frac{(1-\omega)(1-\beta\omega)}{\omega}$$

and $\hat{mc}_t$ is the deviation of real marginal costs around its steady state.

The New Keynesian Phillips curve To obtain the New Keynesian Phillips curve expressed in equation (2.1) it is necessary to show that the deviation of real marginal costs around its steady-state is equivalent to the output gap. When all firms are able to adjust their price every period, hence, $\omega = 0$, then equation (8.21) is reduced to

$$\left(\frac{p_t^\star}{P_t}\right) = \left(\frac{\theta}{\theta-1}\right) mc_t = \alpha mc_t$$

which means that prices are simply a mark-up, α, over current real marginal costs. However, when all firms are able to adjust prices freely, then $p_t^\star = P_t$ and $mc_t = \frac{1}{\alpha}$. Using the definition of the real marginal costs in equation (8.17) this implies

$$\frac{W_t}{P_t} = \frac{Z_t}{\alpha}$$

[66] Consult Walsh (2010) for details on the derivation.

Using further the condition in (8.12) that the household marginal rate of substitution must equal the real wage in equilibrium then yields the following condition

$$\frac{W_t}{P_t} = \frac{Z_t}{\alpha} = \frac{\chi \left(\varpi_t^N\right)^{1-\eta} N_t^{\eta}}{C_t^{-\sigma}} \tag{8.24}$$

The model describes a closed economy with no investments and no government spending. The resource constraint is therefore given by

$$C_t = Y_t$$

The deviation of the resource constraint around steady state is then expressed as follows

$$\hat{c}_t = \hat{y}_t \tag{8.25}$$

which states that all output is consumed. Using (8.25) the log-linearisation of (8.24) around its steady state then gives

$$\hat{y}_t^f = \left(\frac{1+\eta}{\sigma+\eta}\right)\hat{z}_t + \left(\frac{1-\sigma}{\sigma+\eta}\right)\varpi_t^c - \left(\frac{1+\eta}{\sigma+\eta}\right)\varpi_t^n \tag{8.26}$$

where $\hat{y}_t^f$ denotes the deviation of the flexible-price equilibrium output around steady-state. Using (8.25), and deviation of the production function and the condition in (8.12) around steady state then yield the following expression for the deviation of real marginal costs around steady state

$$\hat{mc}_t = (\sigma+\eta)\left[\hat{y}_t - \left(\frac{1+\eta}{\sigma+\eta}\right)\hat{z}_t + \left(\frac{1-\sigma}{\sigma+\eta}\right)\varpi_t^c - \left(\frac{1+\eta}{\sigma+\eta}\right)\varpi_t^n\right] \tag{8.27}$$

Inserting (8.26) into (8.27) then gives

$$\hat{mc}_t = \gamma\left(\hat{y}_t - \hat{y}_t^f\right) \tag{8.28}$$

where $\gamma = (\sigma+\eta)$. Inserting equation (8.28) into (8.23), using $x_t = \hat{y}_t - \hat{y}_t^f$ and allowing for exogenous disturbance, u_t then yields the expression in (2.1)

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + u_t$$

where $\kappa = \gamma\tilde{\kappa}$.

The aggregate demand curve To obtain the aggregate demand expression in equation (2.2), first the deviation of (8.10) around steady state is found and (8.25) is inserted. This gives

$$\hat{y}_t = E_t\hat{y}_{t+1} - \frac{1}{\sigma}\left(\hat{i}_t - E_t\pi_{t+1}\right) + \frac{\sigma-1}{\sigma}\left(E_t\varpi^C_{t+i} - \varpi^C_t\right)$$

Expressing this in terms of the output gap then yields

$$x_t = E_t x_{t+1} - \frac{1}{\sigma}\left(\hat{i}_t - E_t\pi_{t+1}\right) + g_t$$

where $g_t \equiv \left(E_t\hat{y}^f_{t+1} - \hat{y}^f_t\right) + \frac{\sigma-1}{\sigma}\left(E_t\varpi^C_{t+i} - \varpi^C_t\right)$ can be interpreted as exogenous demand disturbance. This expression is equivalent to the aggregate demand relation in (2.2). Note, however, that $\hat{i}_t$ is formally the deviation of the nominal interest rate on a one-period bond around steady state, but in the analysis of this thesis it is assumed to be interest rate used to set monetary policy by the central bank.

8.2 The discretionary solution

The following derivation borrows from Clarida et al. (1999) and Vestin (2006). The solution to the price level and the output gap under discretion is on the form

$$p_t = \hat{\theta}_1 p_{t-1} + \hat{\theta}_2 u_t \tag{8.29}$$

$$x_t = \hat{\psi}_1 p_{t-1} - \hat{\psi}_2 u_t \tag{8.30}$$

To find the values of the coefficients $\hat{\theta}_1$ and $\hat{\theta}_2$ the optimisation problem in equation (3.2) is solved subject to the Phillips curve in equation (3.3). Solving (3.2) yields the following first-order condition

$$x_t = -\frac{\kappa}{\lambda}\pi_t \tag{8.31}$$

Substituting equation (8.31) into equation (2.1) to eliminate x_t then gives

$$\pi_t = \frac{\beta}{1+\frac{\kappa^2}{\lambda}}E_t\pi_{t+1} + \frac{1}{1+\frac{\kappa^2}{\lambda}}u_t \tag{8.32}$$

Under discretion $\hat{\theta}_1 = 1$. Using this equation (8.29) then implies that the solution to inflation is on the form

$$\pi_t = \hat{\theta}_2 u_t \tag{8.33}$$

Leading the solution one period and taking expectations then yields

$$E_t\pi_{t+1} = \hat{\theta}_2 E_t u_{t+1} = \hat{\theta}_2\rho u_t \tag{8.34}$$

using the definition of u_t from section 2.1.

$\hat{\theta}_2$ is then found by inserting (8.33) and (8.34) in (8.32). Rearranging the resulting expression then gives

$$\hat{\theta}_2 = \frac{\lambda}{\kappa^2 + \lambda(1-\beta\rho)} \tag{8.35}$$

To find $\hat{\psi}_1$ and $\hat{\psi}_2$ it is convenient to reformulate equation (2.1) expressed in terms of the price level

$$x_t = \frac{1}{\kappa}(1+\beta)p_t - \frac{\beta}{\kappa}E_t p_{t+1} - \frac{1}{\kappa}p_{t-1} - \frac{1}{\kappa}u_t \tag{8.36}$$

Using the solution found above leading equation (8.29) one period forward and taking expectations yields

$$E_t p_{t+1} = p_t + \hat{\theta}_2 \rho u_t \tag{8.37}$$

Substituting (8.29) and (8.37) into (8.36) then gives

$$x_t = -\hat{\psi}_2 u_t$$

where

$$\hat{\psi}_1 = 0$$

$$\hat{\psi}_2 = \frac{\kappa}{\lambda}\hat{\theta}_2 = \frac{\kappa}{\kappa^2 + \lambda(1-\beta\rho)}$$

8.3 The optimal commitment solution

The following derivation borrows from Clarida et al. (1999) and Vestin (2006). The solutions to the price level and the output gap under the optimal commitment solution are given by

$$p_t = \tilde{\theta}_1 p_{t-1} + \tilde{\theta}_2 u_t \tag{8.38}$$

$$x_t = -\tilde{\psi}_1 p_{t-1} - \tilde{\psi}_2 u_t \tag{8.39}$$

To find the solutions to $\tilde{\theta}_1$ and $\tilde{\theta}_2$ first consider the Lagrangian in equation (3.9). The optimal paths for monetary policy are solutions to

$$\frac{\partial \mathcal{L}}{\partial \pi_t} = 0, \; t \geq 0 \tag{8.40}$$

$$\frac{\partial \mathcal{L}}{\partial x_t} = 0, \; t \geq 0 \tag{8.41}$$

The solution to (8.40) is given by

$$\pi_t = -\phi_t \tag{8.42}$$

$$\pi_{t+i} = -\left(\phi_{t+i} - \phi_{t+i-1}\right), \; i \geq 1 \tag{8.43}$$

and the solution to (8.65) is given by

$$\phi_{t+i} = \frac{\lambda}{\kappa} x_{t+i},\ i \geq 0 \tag{8.44}$$

Combining the first-order conditions in (8.42), (8.43) and (8.44) yields

$$\pi_t = -\frac{\lambda}{\kappa} x_t \tag{8.45}$$

$$\pi_{t+i} = -\frac{\lambda}{\kappa} \left(x_{t+i} - x_{t+i-1} \right), \quad \forall i \geq 1 \tag{8.46}$$

Substituting (8.46) into equation (2.1) in order to eliminate inflation then leads to the following second-order stochastic difference equation

$$x_t = \theta x_{t-1} + \theta \beta E_t x_{t+1} - \theta \frac{\kappa}{\lambda} u_t \tag{8.47}$$

which is rewritten on the form

$$E_t x_{t+1} - \frac{1}{\theta\beta} x_t + \frac{1}{\beta} x_{t-1} = \frac{\kappa}{\beta\lambda} u_t \tag{8.48}$$

The solution to the difference equation is found by solving

$$h^2 - \frac{1}{\theta\beta} h + \frac{1}{\beta} = 0$$

which has one stable root given by $h = \frac{1-\sqrt{1-4\beta\theta^2}}{2\theta\beta} \in (0,1)$ and one unstable root given by $h_2 = \frac{1}{\beta h}$. It is then possible to rewrite (8.48) as

$$(1 - hL) \left(1 - \frac{1}{\beta\theta} L \right) x_{t+1} = \frac{\kappa}{\beta\lambda} u_t$$

and solving this yields

$$x_t = h x_{t-1} - \frac{\kappa h}{\lambda(1 - h\beta\rho)} u_t \tag{8.49}$$

Substituting (8.49) back into (8.46) gives an expression for inflation

$$\pi_t = h \pi_{t-1} + \frac{h}{1 - h\beta\rho} (u_t - u_{t-1})$$

or put in terms of the price level using $\pi_t = p_t - p_{t-1}$

$$p_t = \tilde{\theta}_1 p_{t-1} + \tilde{\theta}_2 u_t \tag{8.50}$$

where

$$\tilde{\theta}_1 = \frac{\left(\lambda\left(1+\beta\right)+\kappa^2\right)\left(1-\sqrt{1-4\beta\left(\frac{\lambda}{\lambda(1+\beta)+\kappa^2}\right)^2}\right)}{2\lambda\beta}$$

$$\tilde{\theta}_2 = \frac{\tilde{\theta}_1}{1-\beta\tilde{\theta}_1\rho}$$

Equation (8.49) already presents an expression for x_t, however, to get x_t on the form in (8.39) and find $\tilde{\psi}_1$ and $\tilde{\psi}_2$ it is convenient to consider equation (2.1) expressed in terms of the price level

$$x_t = \frac{1}{\kappa}(1+\beta)p_t - \frac{\beta}{\kappa}E_t p_{t+1} - \frac{1}{\kappa}p_{t-1} - \frac{1}{\kappa}u_t \tag{8.51}$$

Inserting (8.50) and the expectations to the one period lead of (8.50) in (8.51) then yields

$$x_t = -\tilde{\psi}_1 p_{t-1} - \tilde{\psi}_2 u_t$$

where

$$\tilde{\psi}_1 = \frac{\left(1-\tilde{\theta}_1\beta\right)\left(1-\tilde{\theta}_1\right)}{\kappa}$$

$$\tilde{\psi}_2 = \frac{1-\tilde{\theta}_2\left[1+\beta\left(1-\rho-\tilde{\theta}_1\right)\right]}{\kappa}$$

8.4 Variance calculation

The variance of inflation and the output gap used in sections 3 and 4 are calculated for the price level and the output gap on the form

$$p_t = \theta_1 p_{t-1} + \theta_2 u_t \tag{8.52}$$

$$x_t = -\psi_1 p_{t-1} - \psi_2 u_t \tag{8.53}$$

as follows from section 3.

An expression for inflation then follows from equation (8.52) by subtracting p_{t-1} on both sides of the equation

$$\pi_t = -(1-\theta_1)p_{t-1} + \theta_2 u_t \tag{8.54}$$

The variance of inflation can then be calculated using equation (8.54)

$$Var(\pi_t) = (1-\theta_1)^2 Var(p_{t-1}) + \theta_2^2 Var(u_t) - 2(1-\theta_1)\theta_2 Cov(p_{t-1}, u_t) \tag{8.55}$$

To simplify the expression in equation (8.55) $Var(p_{t-1})$ is needed. This can be found from equation (8.52). Stationarity implies $Var(p_t) = Var(p_{t-1})$. Hence, it suffices to calculate

$Var(p_t)$

$$Var(p_t) = \theta_1^2 Var(p_{t-1}) + \theta_2^2 Var(u_t) + 2\theta_1\theta_2 Cov(p_{t-1}, u_t)$$

using stationarity to simplify

$$Var(p_t) = \frac{\theta_2^2}{(1-\theta_1^2)} Var(u_t) + \frac{2\theta_1\theta_2}{(1-\theta_1^2)} Cov(p_{t-1}, u_t) \tag{8.56}$$

where

$$Cov(p_{t-1}, u_t) = Cov(\theta_1 p_{t-2} + \theta_2 u_{t-1}, \rho u_{t-1} + \varepsilon_t)$$

using the one period lagged version of equation (8.52), the expression for u_t from section 2.1 gives

$$Cov(p_{t-1}, u_t) = \theta_1 \rho Cov(p_{t-2}, u_{t-1}) + \theta_2 \rho Var(u_{t-1})$$

and using the stationarity property yields

$$Cov(p_{t-1}, u_t) = \frac{\theta_2 \rho}{(1-\theta_1\rho)} \sigma_u^2 \tag{8.57}$$

Substituting equation (8.57) into (8.56) and using the definition of u_t from section 2.1then gives

$$Var(p_t) = \frac{\theta_2^2(1+\theta_1\rho)}{(1-\theta_1^2)(1-\theta_1\rho)} \sigma_u^2 \tag{8.58}$$

And then a simplified version of (8.55) follows by using (8.57) and (8.58)

$$Var(\pi_t) = (1-\theta_1)^2 \frac{\theta_2^2(1+\theta_1\rho)}{(1-\theta_1^2)(1-\theta_1\rho)} \sigma_u^2 + \theta_2^2\sigma_u^2 - 2(1-\theta_1)\theta_2 \frac{\theta_2\rho}{(1-\theta_1\rho)} \sigma_u^2$$

which can be reduced to

$$Var(\pi_t) = \xi_1^2 \sigma_u^2 \tag{8.59}$$

where $\xi_1^2 = \frac{2\theta_2^2(1-\rho)}{(1-\theta_1\rho)(1+\theta_1)}$

The variance of the output gap follows from equation (8.53)

$$Var(x_t) = \psi_1^2 Var(p_{t-1}) + \psi_2^2 Var(u_t) - 2\psi_1\psi_2 Cov(p_{t-1}, u_t) \tag{8.60}$$

Using equations (8.57) and (8.58) in (8.60) yields

$$Var(x_t) = \psi_1^2 \frac{\theta_2^2(1+\theta_1\rho)}{(1-\theta_1^2)(1-\theta_1\rho)} \sigma_u^2 + \psi_2^2\sigma_u^2 - 2\psi_1\psi_2 \frac{\theta_2\rho}{(1-\theta_1\rho)} \sigma_u^2$$

which can be reduced to

$$Var(x_t) = \xi_2^2 \sigma_u^2 \tag{8.61}$$

where $\xi_2^2 = \frac{\theta_2^2\psi_1^2(1+\theta_1\rho)+\psi_2^2\left(1-\theta_1^2\right)(1-\theta_1\rho)+2\rho\theta_2\psi_1\psi_2\left(1-\theta_1^2\right)}{\left(1-\theta_1^2\right)(1-\theta_1\rho)}$

8.5 Solution under price level targeting

The derivations to the solution for monetary policy under price level targeting follows Vestin (2006). The solution is on the form

$$p_t = \bar{\theta}_1 p_{t-1} + \bar{\theta}_2 u_t \tag{8.62}$$

$$x_t = -\bar{\psi}_1 p_{t-1} - \bar{\psi}_2 u_t \tag{8.63}$$

To find the coefficients $\bar{\theta}_1$ and $\bar{\theta}_2$ first consider the optimisation problem given by equations (4.3) and (4.4). As opposed to the solution to inflation targeting under discretion derived in appendix 8.2 the optimisation problem now includes two exogenous state variables, p_{t-1} and u_t.

The minimisation problem in (4.3) expresses the central bank's value function which is given by

$$V_t(p_{t-1}, u_t) = E_t\left(\min_{x_t}\left(\frac{1}{2}\left(p_t^2 + \bar{\lambda}x_t^2\right) + \beta V_{t+1}\left(p_t, u_{t+1}\right)\right)\right) \tag{8.64}$$

where the solution is found by solving

$$V_t(p_{t-1}, u_t) = \gamma_0 + \gamma_1 p_{t-1} + \frac{1}{2}\gamma_2 p_{t-1}^2 + \gamma_3 p_{t-1}u_t + \gamma_4 u_t + \frac{1}{2}\gamma_5 u_t^2 \tag{8.65}$$

The guess of the value function is used when taking the conditional expectations in period t of the derivative with respect to p_t. Hence, using (8.65) and the definition of u_t in section 2.1 to find

$$E_t\left(\frac{\partial V_{t+1}\left(p_t, u_{t+1}\right)}{\partial p_t}\right) = \gamma_2 p_t + \gamma_3 \rho u_t \tag{8.66}$$

where γ_1 has been left out because $\bar{x}$, $\bar{\pi}$and $\bar{p}$ are all set to zero and thus there is no drift in the price level. This yields the following guess of the value function

$$V_t(p_{t-1}, u_t) = \frac{1}{2}\gamma_2 p_{t-1}^2 + \gamma_3 p_{t-1}u_t \tag{8.67}$$

Expressing equation (2.1) in terms of the price level gives

$$x_t = \frac{1}{\kappa}(1+\beta)p_t - \frac{\beta}{\kappa}E_t p_{t+1} - \frac{1}{\kappa}p_{t-1} - \frac{1}{\kappa}u_t \tag{8.68}$$

An expression for $E_t p_{t+1}$ is found by leading (8.62) one period ahead and taking expectations. This can be inserted in (8.68) and the equation thus becomes

$$x_t = \frac{1}{\kappa}(1+\beta(1-\bar{\theta}_1))p_t - \frac{1}{\kappa}p_{t-1} - \frac{1+\beta\rho\bar{\theta}_2}{\kappa}u_t \tag{8.69}$$

and solving for p_t

$$p_t = \frac{\kappa}{1+\beta(1-\bar{\theta}_1)}x_t + \frac{1}{1+\beta(1-\bar{\theta}_1)}p_{t-1} + \frac{1+\beta\rho\bar{\theta}_2}{1+\beta(1-\bar{\theta}_1)}u_t \tag{8.70}$$

From (8.70) the following derivative is found

$$\frac{\partial p_t}{\partial x_t} = \frac{\kappa}{1+\beta(1-\bar{\theta}_1)} \tag{8.71}$$

Solving the minimisation problem in (8.64) yields the following first-order condition

$$E_t\left(p_t\frac{\partial p_t}{\partial x_t} + \bar{\lambda}x_t + \beta\frac{\partial V_{t+1}(p_t, u_{t+1})}{\partial p_t}\frac{\partial p_t}{\partial x_t}\right) = 0 \tag{8.72}$$

Inserting (8.66), (8.70) and (8.71) into (8.72) and rearranging gives

$$p_t = \frac{\lambda\left(1+\beta\left(1-\bar{\theta}_1\right)\right)}{\kappa^2+\lambda\left(1+\beta\left(1-\bar{\theta}_1\right)\right)^2+\beta\kappa^2\gamma_2}p_{t-1} + \frac{\lambda\left(1+\beta\left(1-\bar{\theta}_1\right)\right)\left(1+\beta\rho\bar{\theta}_2\right)-\beta\rho\kappa^2\gamma_3}{\kappa^2+\lambda\left(1+\beta\left(1-\bar{\theta}_1\right)\right)^2+\beta\kappa^2\gamma_2} \tag{8.73}$$

To solve (8.73) the coefficients γ_2 and γ_3 are determined by using

$$\frac{\partial V_t(p_{t-1}, u_t)}{\partial p_{t-1}} = \gamma_2 p_{t-1} + \gamma_3 u_t \tag{8.74}$$

which is the derivative of (8.67) with respect to p_t and the equivalent derivative obtained from (8.64)

$$\frac{\partial V_t(p_{t-1}, u_t)}{\partial p_{t-1}} = E_t\left(-\frac{\bar{\lambda}}{\kappa}x_t\right) \tag{8.75}$$

Inserting (8.62) and (8.68) into (8.75) then yields

$$\frac{\partial V_t(p_{t-1}, u_t)}{\partial p_{t-1}} = \frac{\bar{\lambda}}{\kappa^2}\left(1-\left(1+\beta\left(1-\bar{\theta}_1\right)\right)\bar{\theta}_1\right)p_{t-1} + \frac{\bar{\lambda}}{\kappa^2}\left(\left(1+\beta\rho\bar{\theta}_2\right)-\left(1+\beta\left(1-\bar{\theta}_1\right)\right)\bar{\theta}_2\right)u_t \tag{8.76}$$

Using the envelope theorem on (8.74) and (8.76) gives the following solutions to γ_2 and γ_3

$$\gamma_2 = \frac{\bar{\lambda}}{\kappa^2}\left(1-\left(1+\beta\left(1-\bar{\theta}_1\right)\right)\bar{\theta}_1\right)P \tag{8.77}$$

$$\gamma_3 = \frac{\bar{\lambda}}{\kappa^2}\left(\left(1+\beta\rho\bar{\theta}_2\right)-\left(1+\beta\left(1-\bar{\theta}_1\right)\right)\bar{\theta}_2\right) \tag{8.78}$$

Substituting (8.77) and (8.78) into (8.73) then leads to the solution for the price level given by

$$p_t = \bar{\theta}_1 p_{t-1} + \bar{\theta}_2 u_t \tag{8.79}$$

where

$$\bar{\theta}_1 = \frac{\omega\bar{\lambda}}{\kappa^2 + \omega^2\bar{\lambda} + \beta\bar{\lambda}\left(1 - \omega\bar{\theta}_1\right)}$$

$$\bar{\theta}_2 = \frac{\omega\bar{\lambda} + \beta\rho\bar{\lambda}\left[2\omega\bar{\theta}_2 - \left(1 + \beta\rho\bar{\theta}_2\right)\right]}{\kappa^2 + \omega^2\bar{\lambda} + \beta\bar{\lambda}\left(1 - \omega\bar{\theta}_1\right)}$$

and $\omega = 1 + \beta\left(1 - \bar{\theta}_1\right)$.

To find the value of $\bar{\psi}_1$ and $\bar{\psi}_2$ equation (8.79) and the expectations to the one period lead of (8.79) are inserted in (8.68). This gives the following solution for x_t

$$x_t = -\bar{\psi}_1 p_{t-1} - \bar{\psi}_2 u_t$$

where

$$\bar{\psi}_1 = \frac{\left(1 - \bar{\theta}_1\beta\right)\left(1 - \bar{\theta}_1\right)}{\kappa}$$

$$\bar{\psi}_2 = \frac{1 - \bar{\theta}_2\left[1 + \beta\left(1 - \rho - \bar{\theta}_1\right)\right]}{\kappa}$$

8.6 Limits calculation

First, the limits are calculated for $\tilde{\theta}_1(\lambda)$

The lower limit is given by

$$\lim_{\lambda\to 0}\tilde{\theta}_1(\lambda) = \lim_{\lambda\to 0}\frac{\left(\lambda\left(1+\beta\right)+\kappa^2\right)\left(1 - \sqrt{1 - 4\beta\left(\frac{\lambda}{\lambda(1+\beta)+\kappa^2}\right)^2}\right)}{2\lambda\beta}$$

which can be rewritten to

$$\lim_{\lambda\to 0}\tilde{\theta}_1(\lambda) = \lim_{\lambda\to 0}\frac{1 - \sqrt{1 - 4\beta\left(\frac{\lambda}{\lambda(1+\beta)+\kappa^2}\right)^2}}{\frac{2\lambda\beta}{(\lambda(1+\beta)+\kappa^2)}}$$

Setting $s = \frac{\lambda}{(\lambda(1+\beta)+\kappa^2)}$ and using L'Hôpital's rule then yields

$$\lim_{\lambda\to 0}\tilde{\theta}_1(\lambda) = \lim_{\lambda\to 0}\frac{1 - \sqrt{1 - 4\beta s^2}}{2\beta s} = 0$$

The upper limit is given by

$$\lim_{\lambda\to\infty}\tilde{\theta}_1(\lambda) = \lim_{\lambda\to\infty}\frac{\left(\lambda\left(1+\beta\right)+\kappa^2\right)\left(1 - \sqrt{1 - 4\beta\left(\frac{\lambda}{\lambda(1+\beta)+\kappa^2}\right)^2}\right)}{2\lambda\beta}$$

which gives

$$\lim_{\lambda\to\infty}\tilde{\theta}_1(\lambda)=\frac{(1+\beta)\left(1-\sqrt{1-4\beta\frac{1}{(1+\beta)^2}}\right)}{2\beta}=1$$

Second, the limits are calculated for $\bar{\theta}_1(\lambda)$

The lower limit is given by

$$\lim_{\lambda\to 0}\bar{\theta}_1(\lambda)=\lim_{\lambda\to 0}\frac{\omega\bar{\lambda}}{\kappa^2+\omega^2\bar{\lambda}+\beta\bar{\lambda}\left(1-\omega\bar{\theta}_1\right)}=0$$

The upper limit is given by

$$\lim_{\lambda\to\infty}\bar{\theta}_1(\lambda)=\lim_{\lambda\to\infty}\frac{\omega\bar{\lambda}}{\kappa^2+\omega^2\bar{\lambda}+\beta\bar{\lambda}\left(1-\omega\bar{\theta}_1\right)}$$

which gives

$$\lim_{\lambda\to\infty}\bar{\theta}_1(\lambda)=\frac{\omega}{\omega^2+\beta\left(1-\omega\lim_{\lambda\to\infty}\bar{\theta}_1\right)}$$

where $\lim_{\lambda\to\infty}\bar{\theta}_1(\lambda)=1$is a solution.

8.7 Links to central bank websites

All websites are last accessed on 8 September 2011.

Reserve Bank of Australia - www.rba.gov.au

http://www.rba.gov.au/monetary-policy/framework/stmt-conduct-mp-5-30092010.html

Bank of Canada - www.bankofcanada.ca

http://www.bankofcanada.ca/2006/11/press-releases/joint-statement-government-canada/

http://www.bankofcanada.ca/about/backgrounders/inflation-control-target-2/

European Central Bank - www.ecb.europa.eu

http://www.ecb.europa.eu/mopo/strategy/pricestab/html/index.en.html

Bank of Japan - www.boj.or.jp/en

http://www.boj.or.jp/en/mopo/outline/index.htm/

http://www.boj.or.jp/en/mopo/outline/sgp.htm/

http://www.boj.or.jp/en/announcements/release_2001/k010319a.htm/

Reserve Bank of New Zealand - www.rbnz.govt.nz

http://www.rbnz.govt.nz/monpol/pta/3517828.html

Norges Bank - www.norges-bank.no/en

http://www.norges-bank.no/en/about/mandate-and-core-responsibilities/legislation/regulation-on-monetary-policy/

Riksbank - www.riksbank.com

http://www.riksbank.com/templates/Page.aspx?id=10543

Swiss National Bank - www.snb.ch/en

http://www.snb.ch/en/iabout/monpol/id/monpol_strat/8

Federal Reserve - www.federalreserve.gov

Bank of England - www.bankofengland.co.uk

http://www.bankofengland.co.uk/monetarypolicy/framework.htm